DAILY OPERATIONAL TOOL FOR MAXIMUM BENEFICIAL USE MANAGEMENT OF SURFACE AND GROUND WATERS IN A BASIN

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ABSTRACT

A two-dimensional ground water model based on numerical-analytical procedures is briefly presented and reviewed. The model incorporates the discrete kernel modeling approach, a scanning subsystem, sequential reinitialization and the capability to compute drawdowns at particular locations rather than only average values over cells to provide new input to standard modeling approaches. The model is tested for accuracy and economy. The model will always give results of at least comparable accuracy with standard finite difference techniques provided care is taken in the selection of the model's variables. In some cases, the developed model provides solutions which are nearer the true solution. The model can also provide solutions with comparable accuracy at a lower cost if judicious selection of the model's input is made.

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RESEARCH OBJECTIVES

The overall objective of the research was the development of a surface-subsurface hydrologic model that: (1) portrays accurately fluid movement, (2) is so cost-effective that it can be used on a daily basis over a very long time-horizon, and (3) is designed so that it can be immediately used for integrated management of a surface-groundwater system.

ACHIEVEMENTS OF CONTRACT

It is not desirable to repeat in this completion report all the results obtained over the past three years and the detailed procedures by which they were obtained. These results and procedures can (or will) be found in two theses(Hyre, 1981; Naab, 1982), and several papers in preparation.

Rather a brief review of the methods of attacks and a sample of results will be given. Generally speaking the thrust of the research has been in the direction of development of new and imaginative methods that will greatly reduce the cost of management studies of conjunctive use of surface and ground waters without significant reduction in accuracy. In this regard the project was successful.

DESCRIPTION OF PROGRAMS AQUISIM

Overview of Model

A new model was developed to determine the response of an aquifer system to various pumping strategies and recharge schemes. It incorporates four features which make it a cost-effective management tool: (1) the model is based on the discrete kernel modeling approach, (2) it incorporates a scanning subsystem, (3) it employs a new, more efficient, sequential reinitialization procedure, and (4) it allows the user to input excitation values as both point and distributed excitations. Features 1, 2, and 3 serve to greatly reduce the expense involved in using the computer model. Feature 4 improves both the accuracy and the usefulness of the model output.

The model has been designed to cost-effectively and realistically simulate in two dimensions the response of an aquifer system to various stresses. Program AQUISIM consist of two computer programs, GENERAT and SIMULAT, which combine the discrete kernel approach, sequential reinitialization, and a scanning subsystem to provide a low-cost alternative to conventional finite difference approaches.

Discrete Kernel Modeling Approach

It is not the intent of this report to delve into the theoretical basis of the discrete kernel approach. This method has been well-documented in the literature already(e.g. Morel-Seytoux and Daly, 1975; Illangasekare and Morel-Seytoux, 1982). The theory shows that the average drawdown response over an area to an average distributed excitation (e.g. recharge from seepage) can be written in the form:

$$\Delta s_c(n) = \sum_{\nu=1}^{n} \overline{\delta}_{ce}\,(n-\nu+1)\,\overline{Q}_e(\nu) \qquad (1)$$

where $\Delta s_c(n)$ is the average change in the drawdown over area c at time n, $\bar{Q}_e(\nu)$ is the average recharge over area e at time ν and $\bar{\delta}_{ce}(\ )$ is the "cell-by-cell discrete kernel" which describes how an excitation over area e affects area c. Assuming that the $\bar{\delta}$-coefficients can be determined, the change in drawdown due to any type of distributed excitation can be calculated quite simply from Eq.(1).

There are other types of excitations and responses beside average responses to average excitations. One might be interested in the average drawdown response to a point excitation (e.g. a pumping or recharging well). This particular response can be written in the form:

$$\Delta s_c(n) = \sum_{\nu=1}^{n} \tilde{\delta}_{cp}(n-\nu+1)\ Q_p(\nu) \qquad (2)$$

where $Q_p(\nu)$ is the pumping rate in well p during time period ν and $\tilde{\delta}_{cp}(\ )$ is the "cell-by-point discrete kernel" which describes how pumping in well p affects the average drawdown over area c.

Since the system is considered linear, these effects are additive. Therefore, if there is more than one recharge plot or one pumping well, the drawdown is found by summing the individual drawdowns:

$$\Delta s_c(n) = \sum_{e=1}^{E} \sum_{\nu=1}^{n} \bar{\delta}_{ce}(n-\nu+1)\ \bar{Q}_e(\nu) + \sum_{p=1}^{P} \sum_{\nu=1}^{n} \tilde{\delta}_{cp}(n-\nu+1)\ Q_p(\nu) \qquad (3)$$

where E is the total number of distributed excitations and P is the total number of point excitations.

Program GENERAT generates all the discrete kernel coefficients necessary to model an aquifer system. Program SIMULAT simulates the response of the system using the coefficients obtained in program GENERAT, to any pattern of excitations.

In all, four types of coefficients are generated. The cell-by-cell discrete kernels, $\bar{\delta}_{ce}$, describe the average drawdown response in area "c" to a uniformly distributed excitation acting over area "e". Examples of excitations of this sort would be seepage from unlined canals, recharge plots, or evapotranspiration. The cell-by-point discrete kernels, $\tilde{\delta}_{cp}$, describe the average drawdown response over area "c" to a point pumping excitation acting at well "p". The only example of excitations of this sort would be pumping or recharging wells. The point-by-point discrete kernels, $\hat{\delta}_{wp}$, describe the point drawdown response at point "w" to a pumping excitation at point "p". This coefficient is needed to calculate the drawdown in wells rather than just average drawdown values over areas. The redistribution discrete kernels, τ^*_{ce}, describe the average drawdown response over area "c" due to unsteady state initial average conditions over area "e". In other words, it describes how nonequilibrium water tables would evolve over time if the system is not otherwise excited.

Program SIMULAT calculates the average drawdown over prescribed areas and the point drawdowns in the wells using the four coefficients generated in GENERAT. Overall, the average drawdown over area "c" at time n can be written as:

$$\Delta\bar{s}_c(n) = \bar{s}^i_c + \sum_{e=1}^{E} \sum_{\nu=1}^{n} \bar{\delta}_{ce}(n-\nu+1)\, \bar{Q}_e(\nu) + \sum_{p=1}^{P} \sum_{\nu=1}^{n} \tilde{\delta}_{cp}(n-\nu+1)\, Q_p(\nu)$$

$$- \sum_{q=1}^{N} \tau^*_{cq}(n)\, \bar{s}^i_q \qquad (4)$$

where $\bar{s}^i_c$ is the initial average drawdown over area c and N is the total number of areas in the system.

The point drawdowns in a well are due to the same three excitations. However, the discrete kernel used to describe the response to the point excitation is the $\hat{\delta}$-coefficient rather than the $\tilde{\delta}$-coefficient. Therefore, the drawdown at point "w" is:

$$s_w(n) = s_w^i + \sum_{e=1}^{E} \sum_{\nu=1}^{n} \bar{\delta}_{we}(n-\nu+1)\, \bar{Q}_e(\nu) + \sum_{p=1}^{P} \sum_{\nu=1}^{n} \hat{\delta}_{wp}(n-\nu+1) Q_p(\nu)$$

$$- \sum_{q=1}^{N} \tau^*_{wq}(n)\, \bar{s}_q^i \qquad (5)$$

Using Eqs.(4) and (5), program SIMULAT calculates the drawdowns over areas and in wells to any pattern of excitations and initial conditions.

Sequential Reinitialization

Program SIMULAT uses the concept of sequential reinitialization to greatly reduce the number of discrete kernels which need to be generated. This procedure helps reduce costs because: (1) only a few periods of discrete kernels need to be generated, (2) the scanning subsystem size can be smaller and therefore less coefficients per excitation area are generated, and (3) the size of storage, both in terms of discrete kernels and excitation rates, can be reduced.

Scanning Subsystem

For any system, a particular excitation has no appreciable effect during a given time span beyond a certain zone of influence. For practical purposes, the effect that an excitation at one end of an aquifer system has on a response point at the other end of the system would be nonexistent for the early periods following the beginning of the excitation. In other words, the discrete kernels describing the response to that excitation would be zero.

For this reason, program GENERAT employs a scanning subsystem in its generation of the discrete kernels. This is a subregion, the size of which is specified by the user, beyond which no discrete kernels are calculated. Thus, for any excitation area or point, the discrete kernels relating the excitation to the response are calculated only for those areas and points lying within the system.

Obviously, selection of the subsystem size is crucial. Since the boundaries around the subsystem are generally assumed to be no-flow, an inappropriate choice of the size could greatly affect the accuracy of the discrete kernels generated. If the subsystem is sized too small, discrete kernels outside of the region will be assumed to be zero when in fact they are not. If the subsystem is sized too large, needless calculations are done.

The size of the subsystem required is a function of the number of periods of discrete kernel generation, the transmissivity and effective porosity of the system, the allowable error, etc.

Point Excitations and Responses

Programs AQUISIM include a feature not found in other groundwater models. Most simulation models require that the excitations for a particular cell be lumped together and the net stress be applied evenly over the area. Likewise, the drawdown response is calculated as an average value for the cell.

By incorporating the point-bycell and point-by-point discrete kernels, programs AQUISIM allow the user to input excitations at a point (i.e. a specific discharge or recharge well) and compute the drawdowns at a point (i.e. a particular observation hole or well) as well as average values for each cell. The drawdown responses to the point excitations are calculated using an analytical technique.

The point-by-cell and point-by-point discrete kernels improve the accuracy of the model output. In addition they serve to reduce the costs as well. Since the well discharge rates need no longer be lumped together as a single excitation, comparable accuracy can be obtained even when the finite difference grids are sized much larger than normal. Sizing the grids larger reduces the number of cells in the system and therefore reduces costs.

ACCURACY ANALYSIS

Results of tests done to verify the accuracy of the four coefficients calculated in program GENERAT are presented in this chapter.

Cell-by-cell Coefficients

The cell-by-cell discrete kernels, $\overline{\delta}_{ce}(\ )$, describe the average response in cell c to a uniformly distributed unit pulse excitation exerted over cell e. This response, an average one due to distributed excitations, is precisely what results from standard finite difference solution of the partial differential equation describing ground water flow.

The major difference between the approach using these discrete kernels and the standard finite difference method of solution of the equations is that the $\overline{\delta}$-coefficients are calculated only for a specified subsystem region whereas standard finite difference models solve the equations for the entire system. Therefore, a comparison of the drawdown responses as calculated using program GENERAT and those from a standard finite difference method will suffice as a test of accuracy comparable with standard methods.

A standard finite difference model was written for these comparisons. It employs the same method of solution as used in program GENERAT. In the standard finite difference model, program FINIDIF, the linearized Boussinesq equation is written at every node in the aquifer system. In program GENERAT, it is written only at nodes contained within the subsystem.

To test the validity of the subsystem and therefore the $\overline{\delta}$-coefficients, the system shown in Figures 1 and 2 was modeled. The transmissivity of each cell is shown on Figure 3 and the boundary conditions on Figure 4. A constant effective porosity value of 0.17 was assumed for all cells.

	12	27	43	59	75	91	107	123	139	155	171	187	203	219	235	243	249	253
	11	26	42	58	74	90	106	122	138	154	170	186	202	218	234	242	248	252
		25	41	57	73	89	105	121	137	153	169	185	201	217	233	241	247	251
		24	40	56	72	88	104	120	136	152	168	184	200	216	232	240	246	250
		23	39	55	71	87	103	119	135	151	167	183	199	215	231	239	245	
		22	38	54	70	86	102	118	134	150	166	182	198	214	230	238	244	
		21	37	53	69	85	101	117	133	149	165	181	197	213	229	237		
			36	52	68	84	100	116	132	148	164	180	196	212	228	236		
		20	35	51	67	83	99	115	131	147	163	179	195	211	227			
		19	34	50	66	82	98	114	130	146	162	178	194	210	226			
	10	18	33	49	65	81	97	113	129	145	161	177	193	209	225			
	9	17	32	48	64	80	96	112	128	144	160	176	192	208	224			
4	8	16	31	47	63	79	95	111	127	143	159	175	191	207	223			
3	7	15	30	46	62	78	94	110	126	142	158	174	190	206	222			
2	6	14	29	45	61	77	93	109	125	141	157	173	189	205	221			
1	5	13	28	44	60	76	92	108	124	140	156	172	188	204	220			

Figure 1. Aquifer system with cells numbered

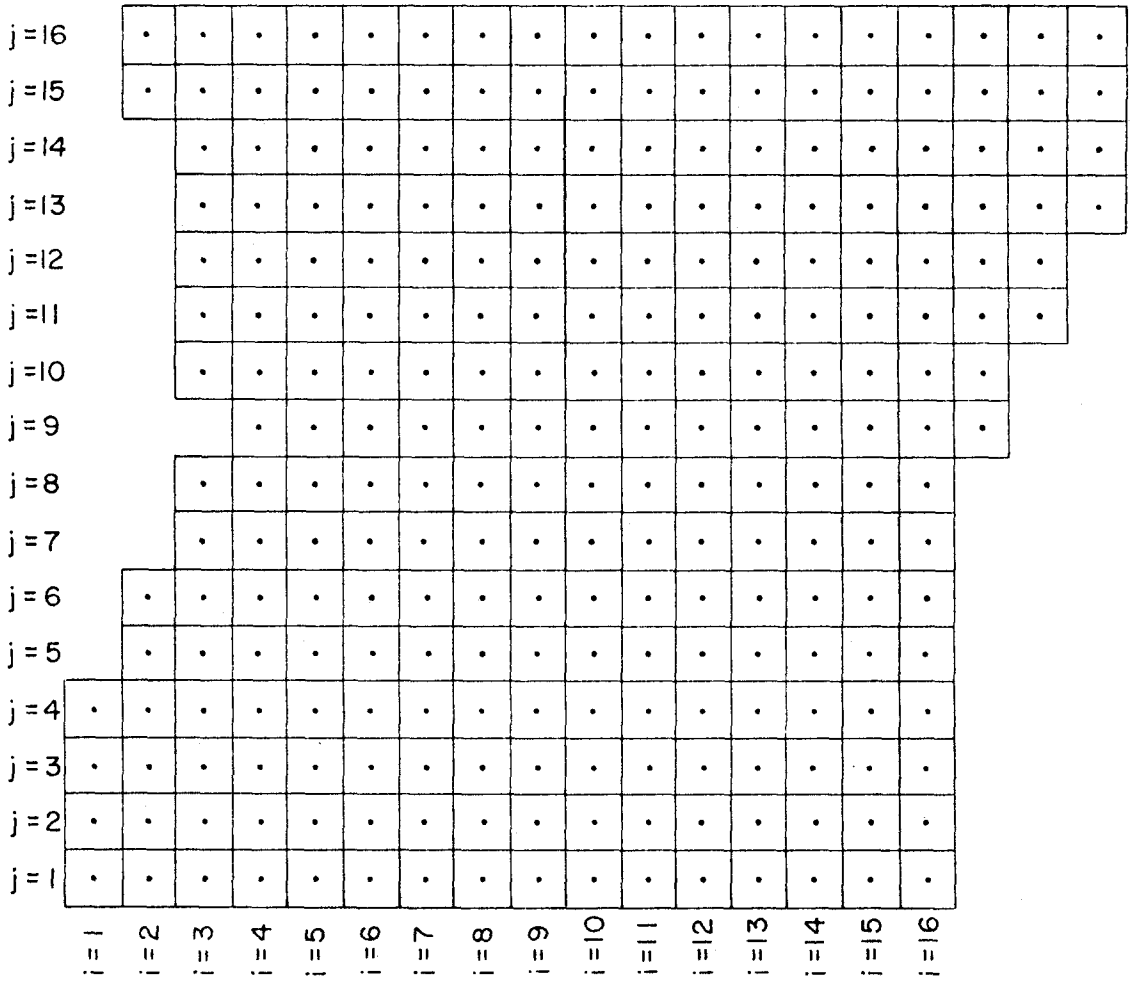

Figure 2. Aquifer system with I-J coordinates

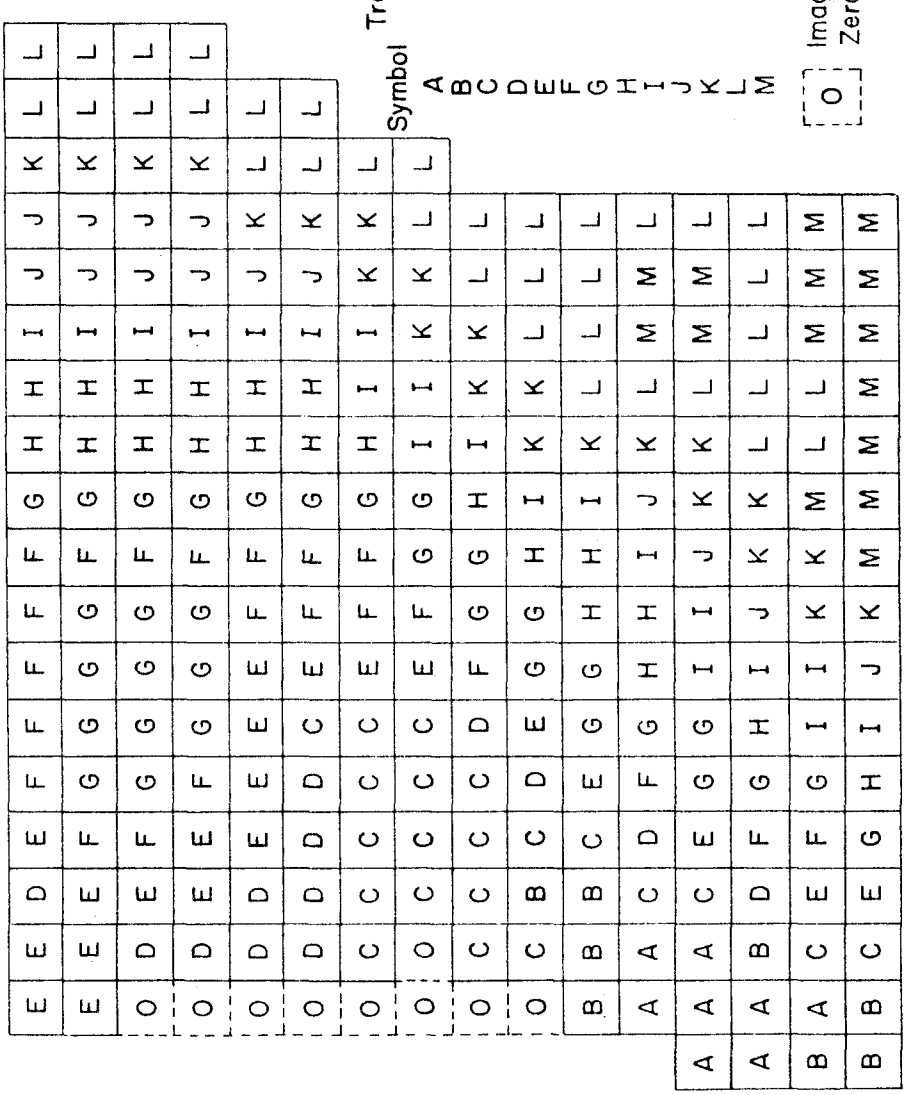

Figure 3. Aquifer system with transmissivity assigned to each cell

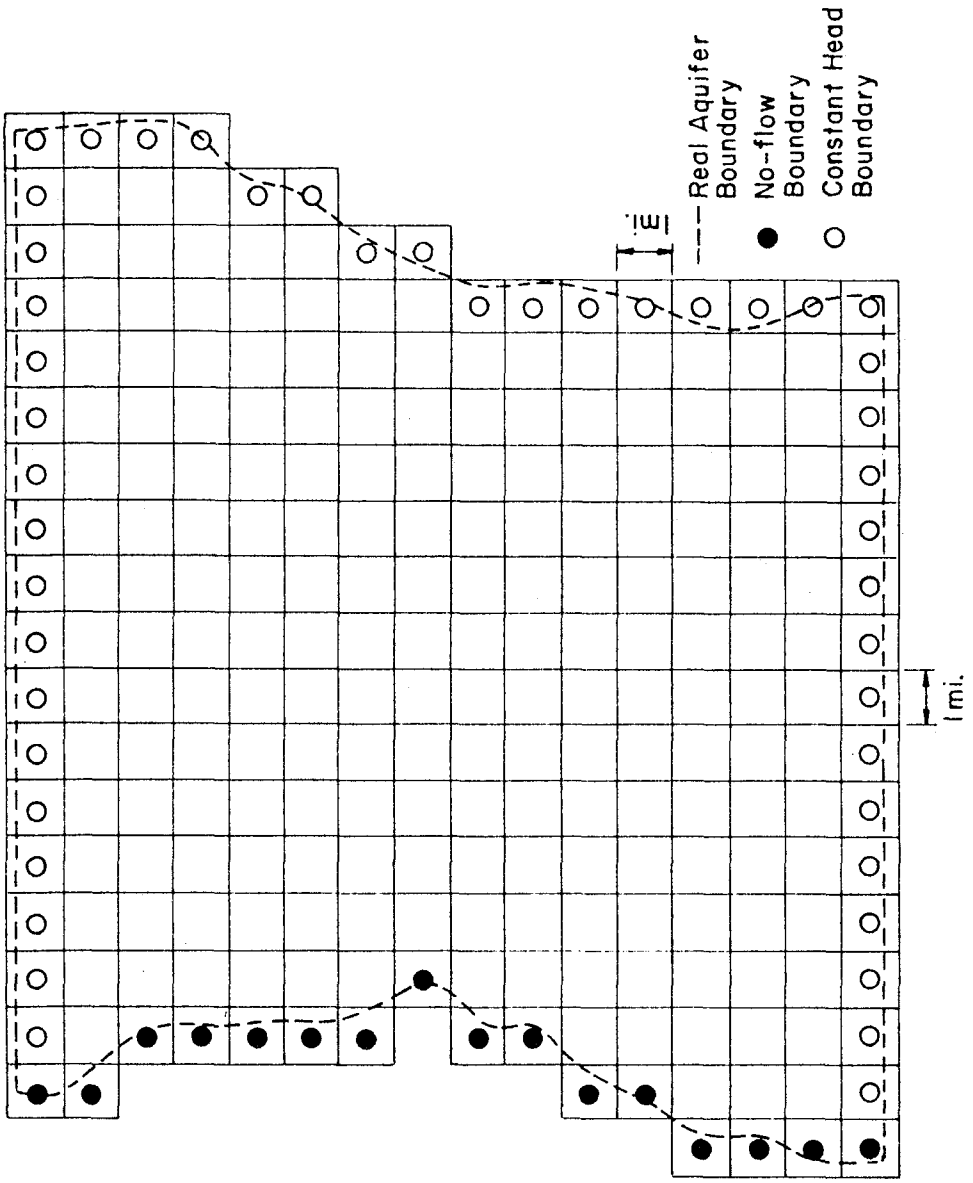

Figure 4. Aquifer system with boundary conditions defined

The water table was taken as initially horizontal and cell #30 was excited with a unit pulse. The response of the system was modeled using programs FINIDIF and GENERAT. A subsystem size of three was chosen for program GENERAT.

Tables 1 and 2 present the results as obtained from programs FINIDIF and GENERAT for times 1 and 10. As can be seen, there is very little difference between the two.

In general, the results of these tests indicates that the subsystem concept employed in program GENERAT is valid and results comparable with standard finite difference methods can be obtained at smaller cost.

Cell-by-point and Point-by-point Coefficients

The cell-by-point coefficients, $\tilde{\delta}_{cp}$, describe the average response in cell c to a unit pulse of pumping at point p. The point-by-point discrete kernels, $\hat{\delta}_{wp}$, describe the response at point w to a unit pulse of pumping at point p. These coefficients are obtained by program GENERAT using analytical procedures.

The $\tilde{\delta}$-coefficients serve two purposes in the simulation procedure. First, they describe the average drawdown in a cell due to a pumping well. The average drawdown is needed in the reinitialization procedure. Second, they provide approximations to the point responses at points lying outside of the cell containing the excitation well. In calculating the point response, program GENERAT only calculates the $\hat{\delta}$-coefficients for observation points lying within the same cell as the excitation well. For points outside of the excitation cell but within the subsystem, the point value is approximated as the average value for the cell (i.e. the $\tilde{\delta}$ is used in place of the $\hat{\delta}$).

In order to test the $\tilde{\delta}$ and $\hat{\delta}$ discrete kernels, they will be used to model the behavior of a system for which an analytical solution is known.

	i=1	i=2	i=3	i=4	i=5	i=6	i=7	i=8	i=9	i=10
j=1										
j=2	.407E-19	.104E-15	.204E-12	.193E-09	.297E-12	.303E-15	.266E-18			
			.204E-12	.193E-09	.297E-12					
j=3	.248E-16	.727E-13	.160E-09	.210E-06	.201E-09	.162E-12	.115E-15	.753E-19		
			.160E-09	.210E-06	.202E-09					
j=4	.300E-19	.778E-16	.156E-12	.172E-09	.255E-12	.268E-15	.227E-18			
			.156E-12	.173E-09	.256E-12					
j=5		.493E-19	.907E-16	.967E-13	.184E-15	.239E-18				
j=6			.426E-19	.410E-16	.909E-19					
j=7				.140E-19						
j=8										

FINIDIF
AQUISIM

Table 1. Cell-by-cell coefficients obtained with programs AQUISIM and FINIDIF - time 1

	i=1	i=2	i=3	i=4	i=5	i=6	i=7	i=8	i=9	i=10
j=1	.187E-17									
j=2	.754E-15	.215E-12	.412E-10	.317E-08	.593E-10	.614E-12	.481E-14	.304E-16	.159E-18	
			.447E-10	.319E-08	.647E-10					
j=3	.514E-13	.148E-10	.266E-08	.199E-06	.331E-08	.323E-10	.231E-12	.136E-14	.692E-17	.151E-19
			.267E-08	.199E-06	.332E-08					
j=4	.560E-15	.161E-12	.316E-10	.285E-08	.512E-10	.544E-12	.410E-14	.254E-16	.120E-18	
			.342E-10	.286E-08	.558E-10					
j=5		.916E-15	.187E-12	.196E-10	.375E-12	.435E-14	.350E-16	.221E-18		
j=6		.405E-17	.787E-15	.843E-13	.167E-14	.213E-16	.185E-18			
j=7			.276E-17	.258E-15	.539E-17	.654E-19				
j=8				.676E-18						

FINIDIF
AQUISIM

Table 2. Cell-by-cell coefficients obtained with programs AQUISIM and FINIDIF - time = 10

For a single well, pumping at a unit pulse rate, located in a homogeneous aquifer of infinite extent and constant thickness, assuming to have negligible vertical flow and no previous development, the drawdown at point w at time ν is (Morel- Seytoux et al, 1973):

$$\hat{\delta}_{wp}(\nu) = \frac{1}{4\pi T}\int_0^{\nu} \left\{ \exp\left[-(\phi R_{wp}^2)/(4T(\nu-\tau)) \right] / (\nu-\tau) \right\} d\tau \qquad (6)$$

where T is the transmissivity and ϕ is the drainable porosity of the aquifer material and R_{wp} is the distance between points w and p.

The solutions to the above equations are compared with the results provided using standard finite difference techniques and program GENERAT. The system model was a finite rectangular aquifer with a length of 15,400 meters and a width of 10,800 meters. The grid layout used in the standard finite difference solution is shown in Figure 5. The layouts used in program GENERAT are shown in Figures 6, 7, 8, and 9. Values of T= 10,000 m^2/week and ϕ=0.20 were used.

Point drawdowns were calculated at distances of 350m, 1050m, 1400m, and 2100m from the pumping well for 16 weeks. In using program GENERAT four runs were made. In run 1, all observation points were contained within the same cell as the excitation well and the grid cell was sized very large (2160m). In run 2, two of the observation wells were placed in an adjacent cell. In run 3, the grid cell was sized the same as that used in the standard finite difference solution and no observation points were placed within the excitation cell. In run 4, the grid was sized twice as large as that used in the standard finite difference solution and only one observation point was contained in the excitation cell. Results using the standard finite difference model were taken from Rodriguez-Amaya (1976).

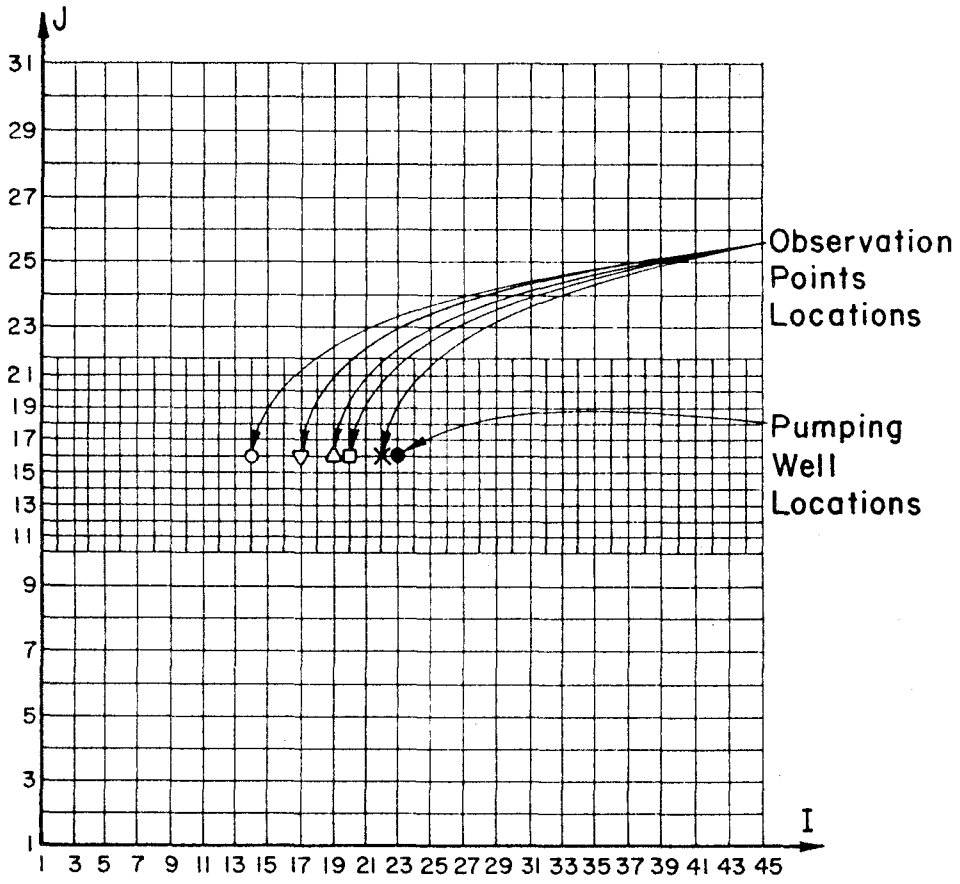

Figure 5. Grid layout used in the standard finite difference solution

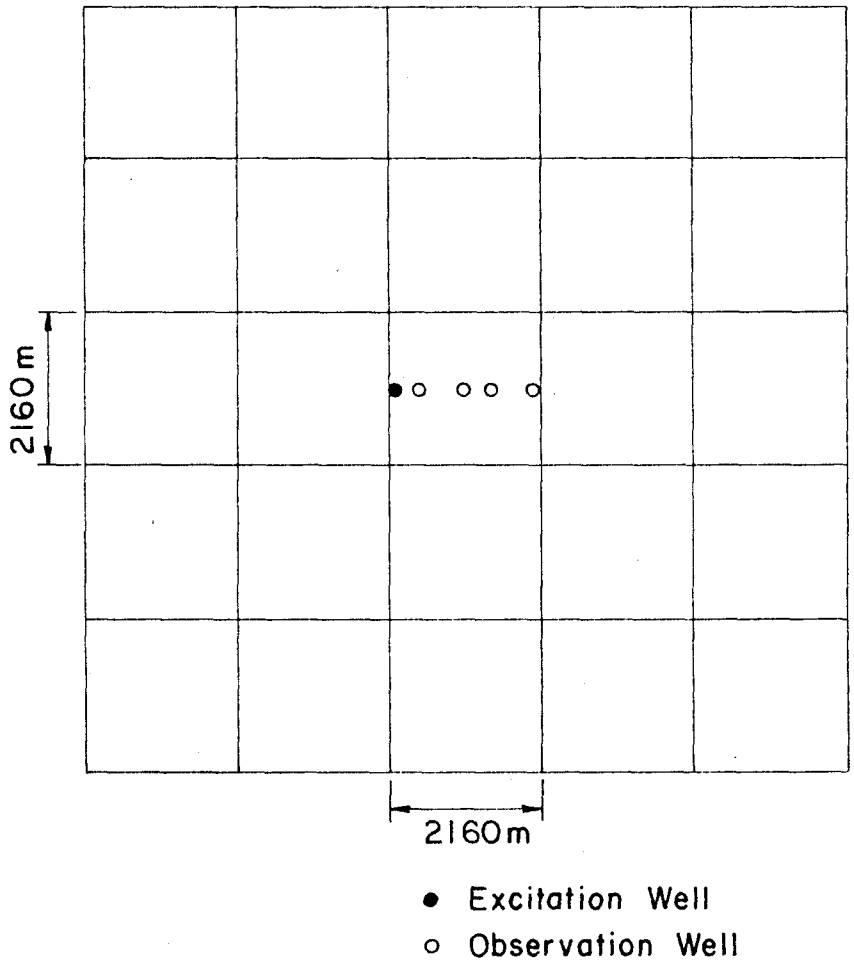

Figure 6. Grid layout used for programs AQUISIM - run 1

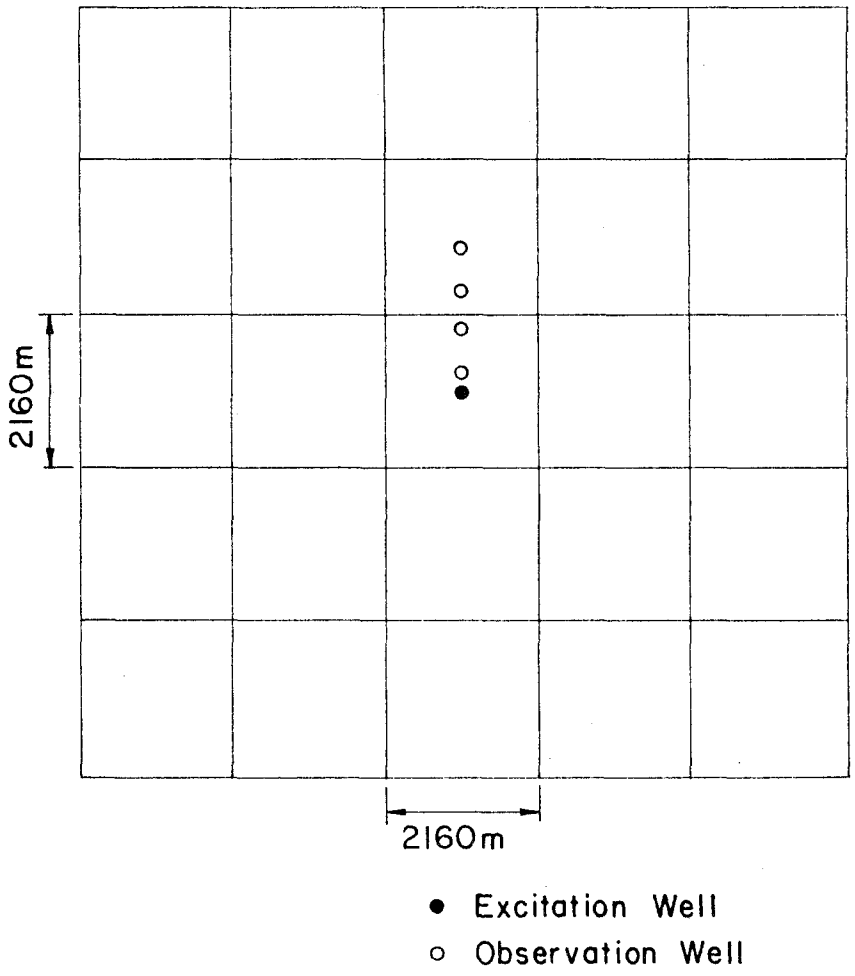

Figure 7. Grid layout used for programs AQUISM - run 2

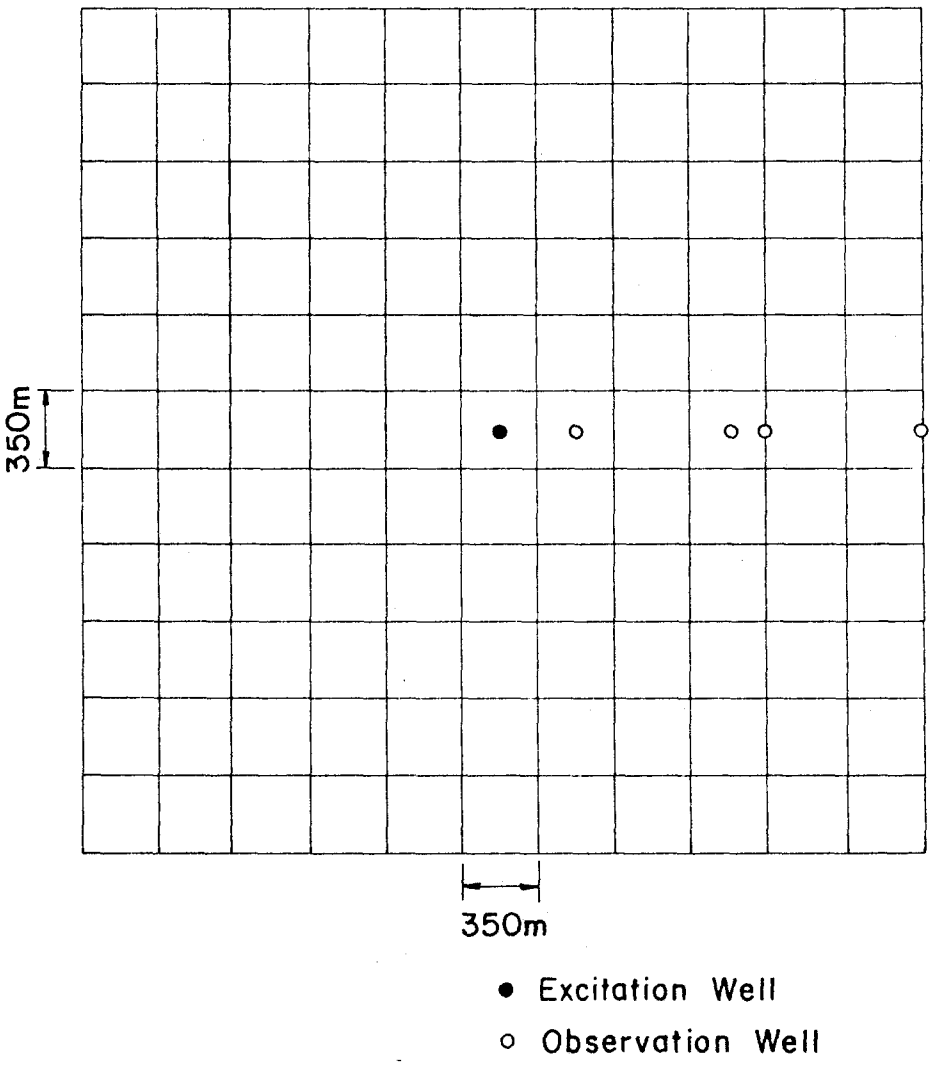

Figure 8. Grid layout used for programs AQUISIM - run 3

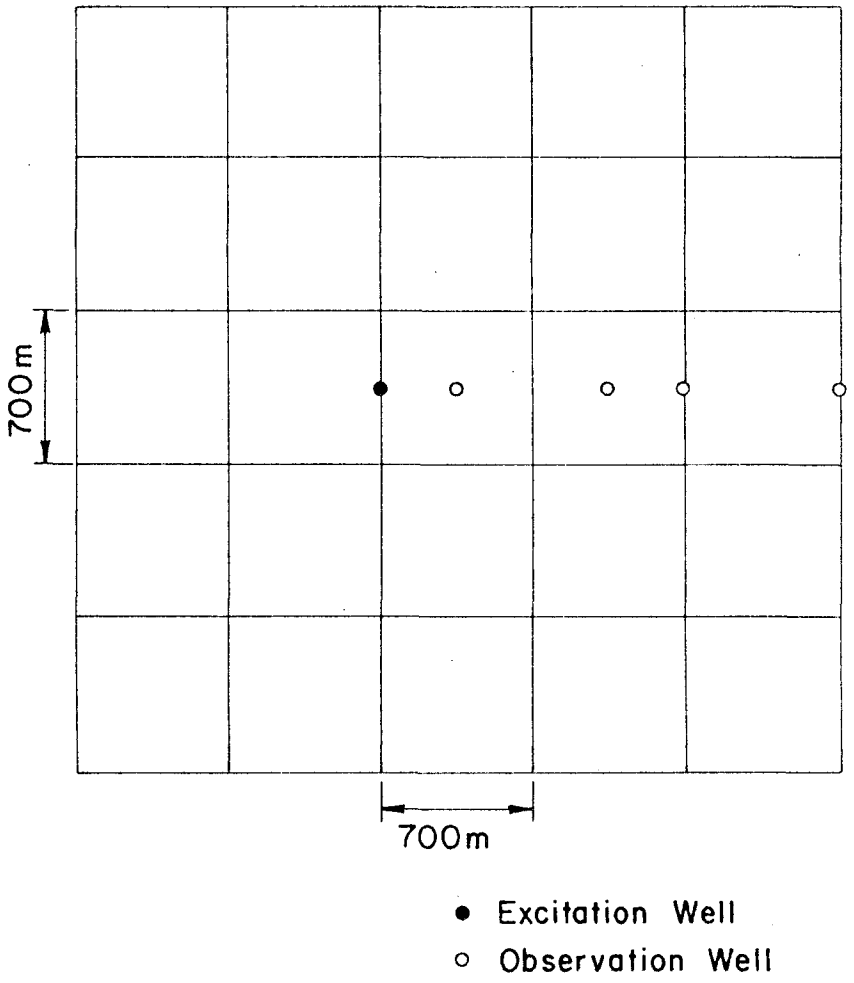

Figure 9. Grid layout used for programs AQUISIM - run 4

Figures 10, 11, 12, and 13 present the drawdown values in m/week-m^3 as obtained from Eq.(6), the standard finite difference model and program GENERAT.

As can be seen from the results of these tests, the accuracy of the solution is quite dependent on both the grid size and the location of the observation points.

Run 1 resulted in solutions which were identical to the analytical one for every observation point. This is due to the fact that the points are all located within the excitation cell and no approximations are made.

Run 2 resulted in identical solutions for the observation points at 350m and 1050m which lie within the excitation cell. However, large error is recognized in the output obtained for the points at 1400m and 2100m. The drawdown for these points is approximated as the average value for the cell and this assumption is obviously not a good one in this case. The grid size is much too large to expect good comparisons. The results for the point at 2100m are much better than those for 1400m. This is largely due to the fact that the 2100m point is closer to the middle of the cell than the 1400m point. The point drawdown is more closely approximated by the average drawdown for points lying near the center of the cell.

The results from Run 3 are in all cases nearly identical to the analytic solution. The initial value of the 350m hydrograph is a bit high but the rest of the values approximate the true solution better than the standard finite difference approach. The good results with this run can be attributed largely to the small grid size (the same size as the finite difference solution).

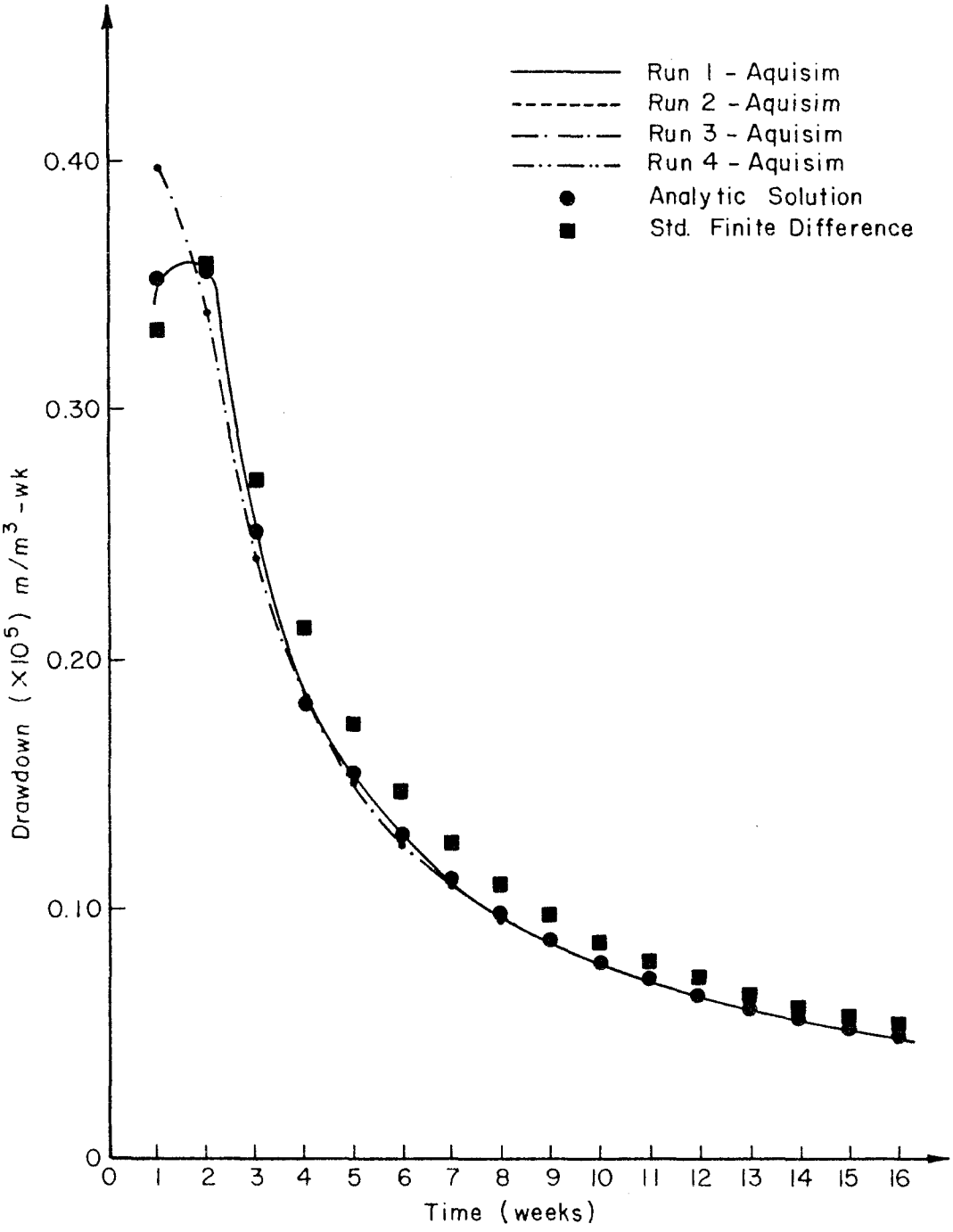

Figure 10. Comparison of drawdowns at 350m from a pumped well in an infinite, homogeneous aquifer

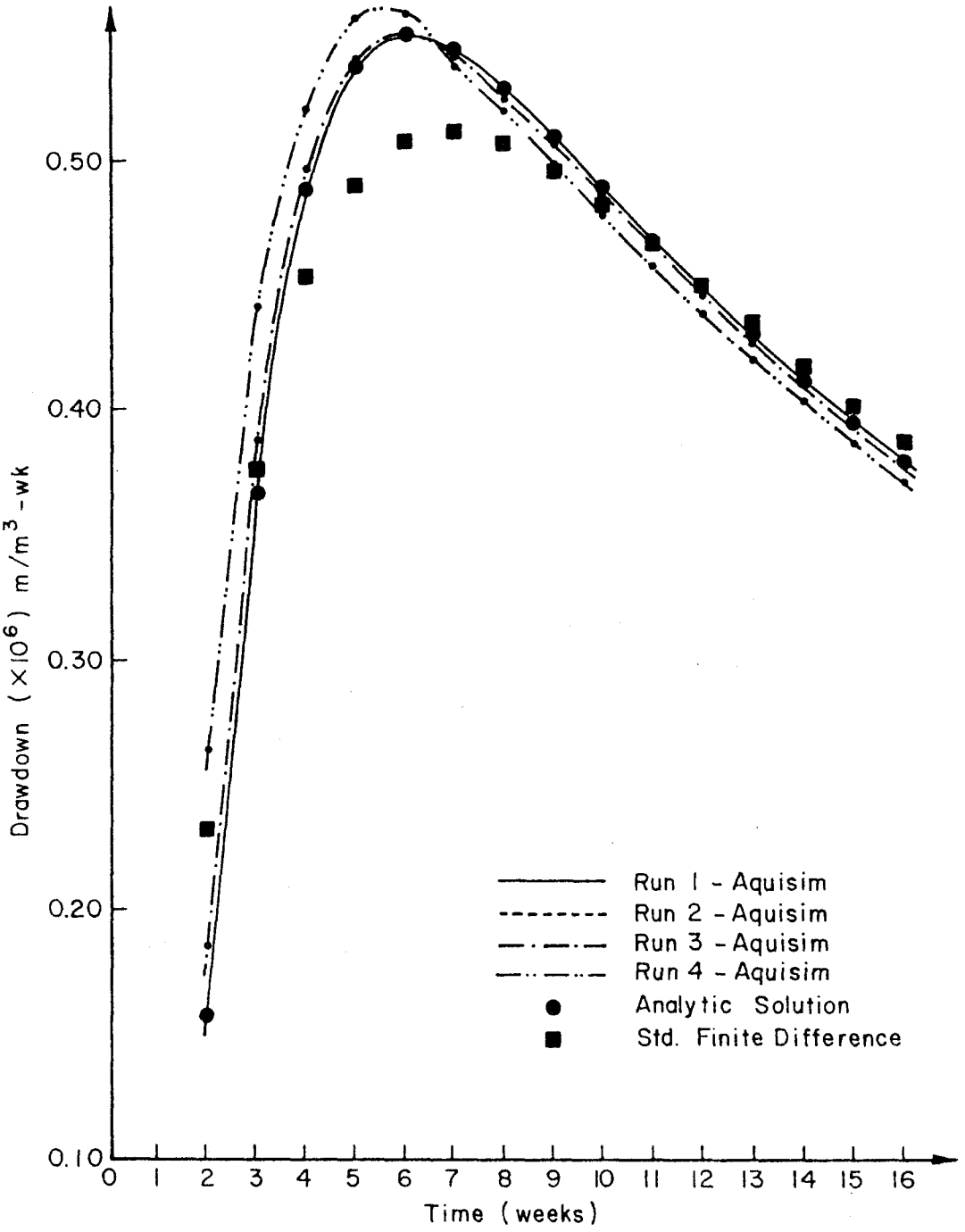

Figure 11. Comparisons of drawdowns at 1050m from a pumped well in an infinite, homogeneous aquifer

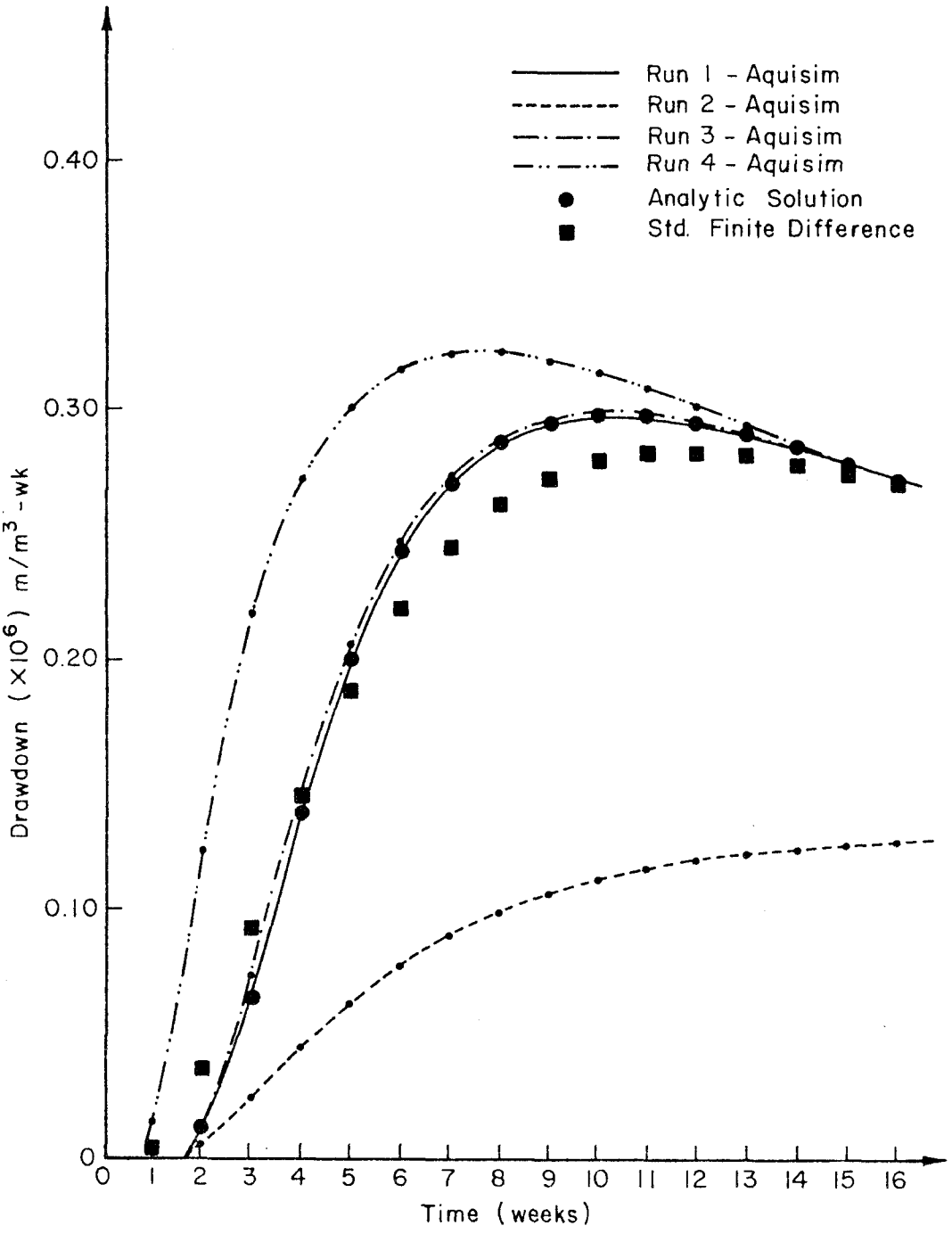

Figure 12. Comparison of drawdowns at 1400m from a pumped well in an infinite, homogeneous aquifer

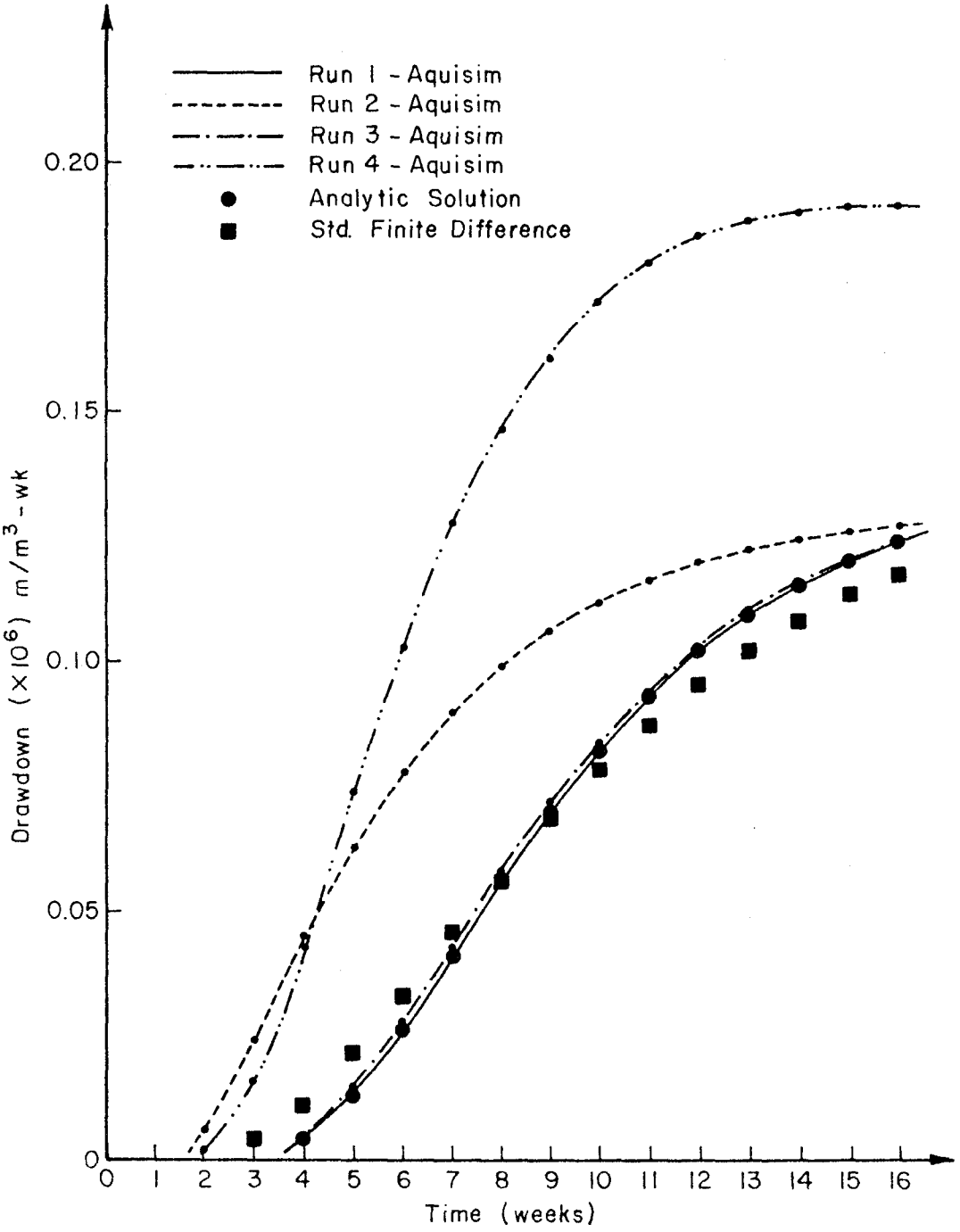

Figure 13. Comparison of drawdowns at 2100m from a pumped well in an infinite, homogeneous aquifer

Run 4 resulted in good results for the 350m and 700m points. The 350m results are identical to the analytic solution since the point lies within the excitation cell. The 1050m results are quite good with a peak value much closer to the real one than that provided by the standard finite difference solution. The good results can be attributed to the proximity of the point to the center of the cell. Both the 1400m and 2100m points showed poor results. This is due to the fact that these points lie on the edge of the cell and replacing the $\tilde{\delta}$ with the $\hat{\delta}$ coefficient is not valid.

In general, the results show that better results can be obtained with programs AQUISIM than with standard finite difference solutions provided that care is taken in the placement of the observation points and the selection of the grid size. Ideally, the observation points should be placed within the same cell as the excitation well. However, if that is not possible, the points should be positioned as close to the center of the cell as possible to obtain the best comparison between the $\tilde{\delta}$ and $\hat{\delta}$ values. If these guidelines are followed, comparable accuracy can be expected even with a much larger grid size.

COST COMPARISON USING SAMPLE AQUIFER SYSTEM

The tests conducted and presented indicate that the methods employed by programs AQUISIM are valid and can provide results which in some cases are more accurate than by standard finite difference methods. The purpose of this section is to present results of tests conducted to compare the costs associated with programs AQUISIM and FINIDIF.

Costs Associated with Each Approach

As stated previously, the standard finite difference model which was written for these comparisons, program FINIDIF, follows the same method of setup and solution of the partial differential equation describing groundwater flow as used in the discrete kernel model, program GENERAT. Identical procedures were followed in order that the costs associated with running each model could be directly compared.

Both models use a five-point implicit finite difference approximation to the linearized Boussinesq equation. The time step used in the solution is increased from a user specified minimum to a maximum.

The costs associated with using program FINIDIF result only from the setup and solution of the finite difference equations. All excitations acting on the system during each period are incorporated into the equations as are the initial conditions of the system. If this approach is to be used to simulate a large aquifer system, for example, the finite difference equation must be written at every node in the system and the resulting system of equations solved for the drawdowns at the end of the time step. This method requires the solution of a large system of equations for every time step. The cost of solution is

dependent only on the number of nodes in the system and the length of simulation. The number of excitations acting on the system (e.g. wells, recharge plots) is not a factor.

The total cost of programs AQUISIM is made up of two parts. The first costs are those associated with generating the discrete kernel coefficients (program GENERAT) and the second costs are those due to simulating the response of the system to a specific pattern of excitations and initial conditions (program SIMULAT). For any particular system, the discrete kernels are generated for all cells or points lying in the same subsystem as an excitation (e.g. well, recharge plot). If, however, sequential reinitialization is to be employed or the effects of nonsteady initial conditions considered, every node in the system is viewed as as excitation node and discrete kernels generated accordingly. In general, this is the case and therefore the coefficients are generated for every cell in the system for the length of the reinitialization period.

The difference in cost between FINIDIF and GENERAT lies in the fact that the discrete kernels are generated only for cells contained in the same subsystem as the excitation whereas program FINIDIF considers every node in the system. Also, the discrete kernels are generated only for the period used in reinitializing the system.

Once the coefficients are generated program SIMULAT can use them to model the system response to varying excitation patterns and initial conditions using Eqs.(4) and (5). The costs associated with programs AQUISIM are dependent on the number of excitations in the system, the subsystem size, the reinitialization period and the length of simulation.

Costs Associated with Example Simulation

In order to demonstrate the difference in the cost of the two models, the aquifer system shown on Figures 1, 2, 3, 4, 14, 15, and 16 will be

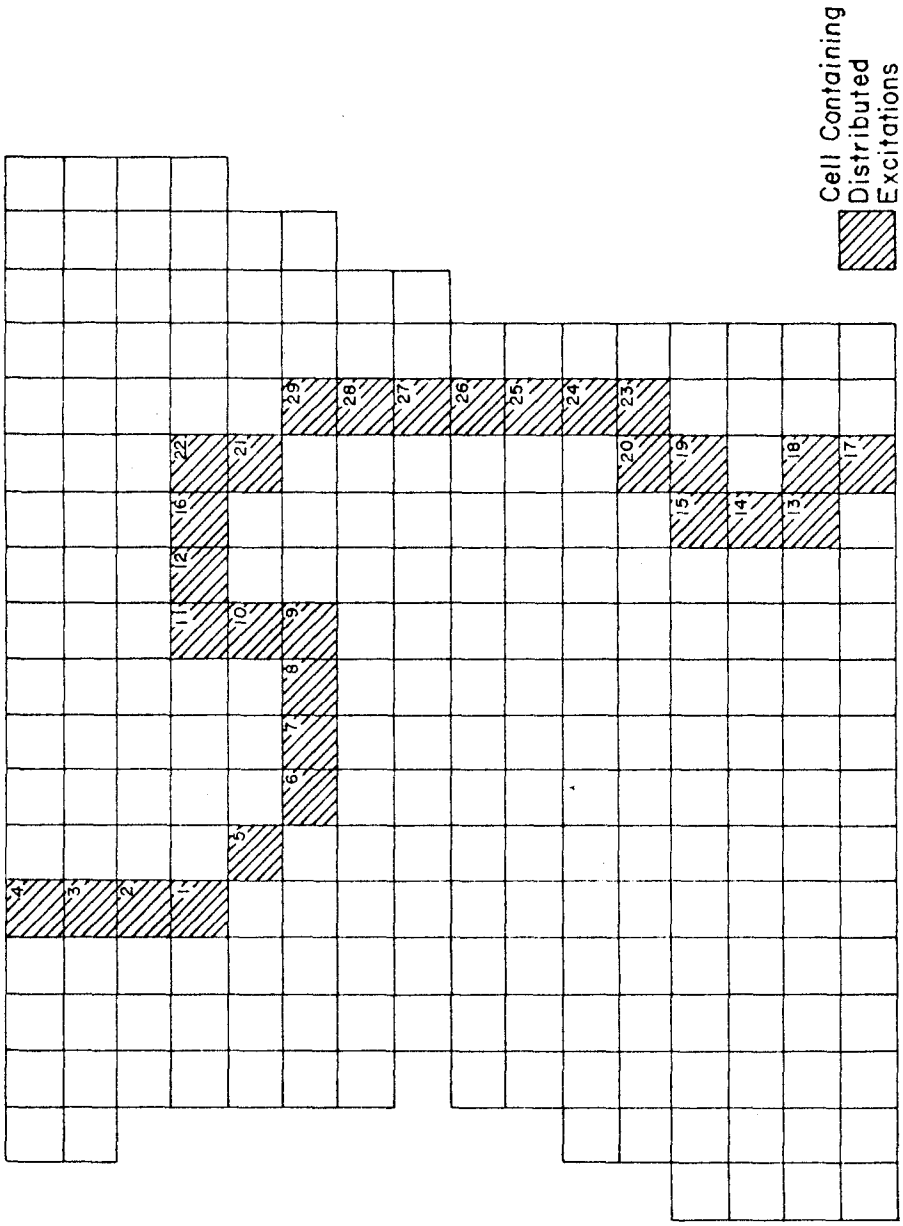

Figure 14. Aquifer system with distributed excitations located

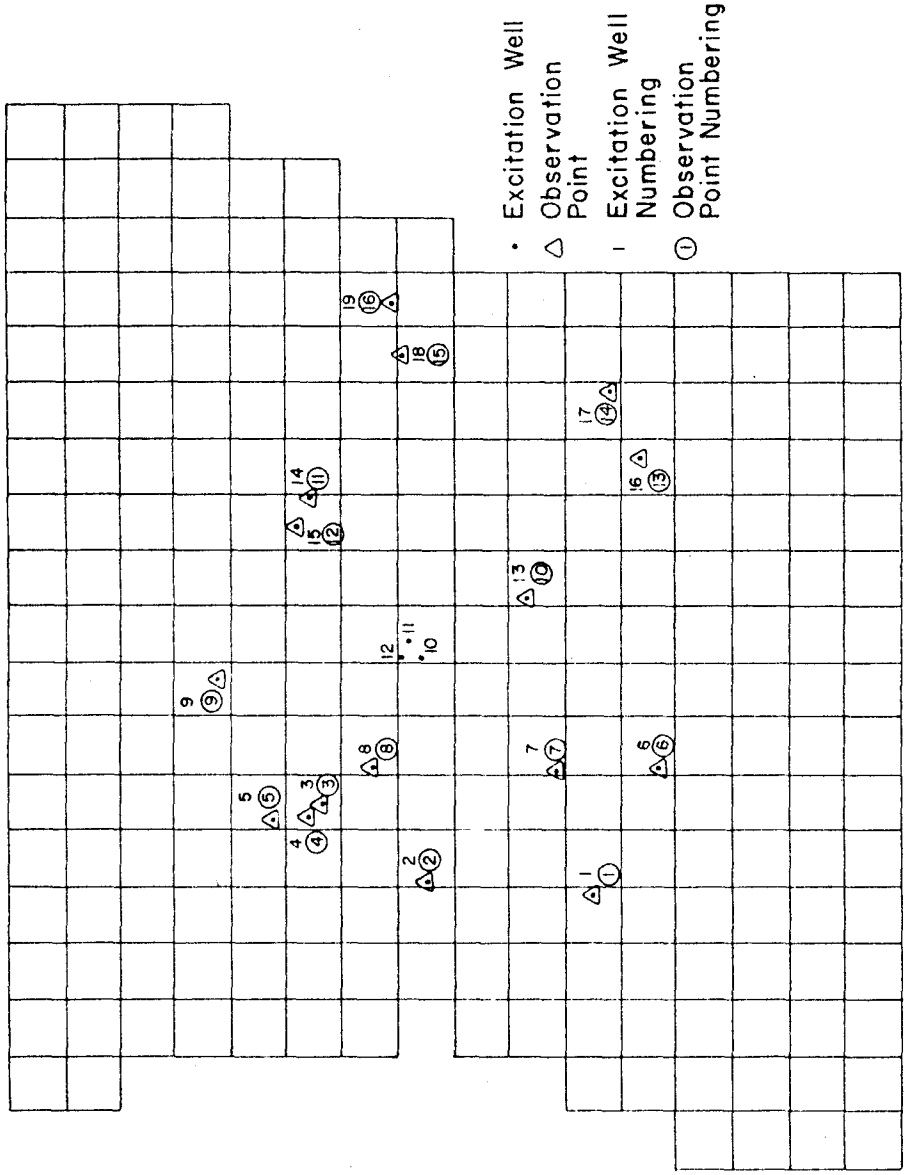

Figure 15. Aquifer system with wells located

	20	30	40	50	60	50	50	90	120	120	100	100	100	125	160	170	185	230
	20	20	30	40	60	60	60	95	120	120	120	110	120	135	160	175	190	230
	0	20	20	40	60	60	70	110	120	120	120	120	130	140	160	175	200	230
	0	20	20	30	60	70	100	100	110	120	120	120	130	140	150	175	200	240
	0	0	0	30	70	90	85	80	80	100	110	110	140	145	160	180	220	
	0	0	0	30	60	90	100	90	85	100	100	110	140	150	160	195	230	
	0	0	10	20	50	75	90	90	90	100	100	110	140	145	180	220		
	0	0	10	20	40	70	80	80	90	100	100	115	145	160	200	240		
	0	10	0	25	45	70	90	100	100	100	100	120	150	170	220			
	0	10	5	30	70	90	100	100	100	100	110	130	165	180	220			
	20	0	20	40	70	100	100	100	100	90	100	130	180	190	220			
	10	10	20	30	65	95	100	100	100	85	120	155	185	200	240			
30	10	20	30	50	65	95	100	100	100	100	140	170	200	220	240			
40	20	30	60	60	60	70	100	100	110	130	160	200	220	240	240			
40	40	60	60	60	60	80	100	100	110	150	170	200	220	230	240			
40	50	60	60	60	60	90	100	100	130	160	180	210	230	240	240			

0 Imaginary Cell with Zero Drawdown

Figure 16. Initial conditions of aquifer system

modeled for variable periods. There are 253 cells in the system and a total of 42 excitation cells, a reasonably sized system. The costs of each program can be briefly summed up as:

(1). Program FINIDIF: The cost will be due to setup and solution of of the finite difference equations. Solution requires solving a 253x253 banded symmetric matrix for each time step.

(2). Programs AQUISIM: The cost will be due to generation of the discrete kernels and then simulating the system. Generation of the discrete kernels requires solution of 253 banded symmetric matrices whose size depends on the subsystem size chosen. These matrices will need to be solved for every time step in the reinitialization period. Simulation requires application of Eq.(4) for every time period.

Five runs were conducted with programs AQUISIM. The subsystem size was taken as three and reinitialization periods of 1, 2, and 3 for the first three runs and a subsystem size of five and reinitialization periods of 1 and 2 for the final runs.

The cost of each program is measured in terms of SRUs. The SRU value associated with a computer run combines the costs of storage, setup and execution of the program. The cost of compilation was not included in the runs conducted. Each SRU unit converts to approximately 7.5 cents of real money when using the Colorado State University Cyber computer.

The cost of using programs FINIDIF and AQUISIM are presented graphically on Figures 17 as costs versus length of simulation. As can be seen, the costs associated with program AQUISIM are dramatically less after approximately the tenth period when a subsystem size of three was chosen. When the subsystem was taken as five, the costs are higher than program FINIDIF up to the 55th period of simulation. Program AQUISIM of course has higher

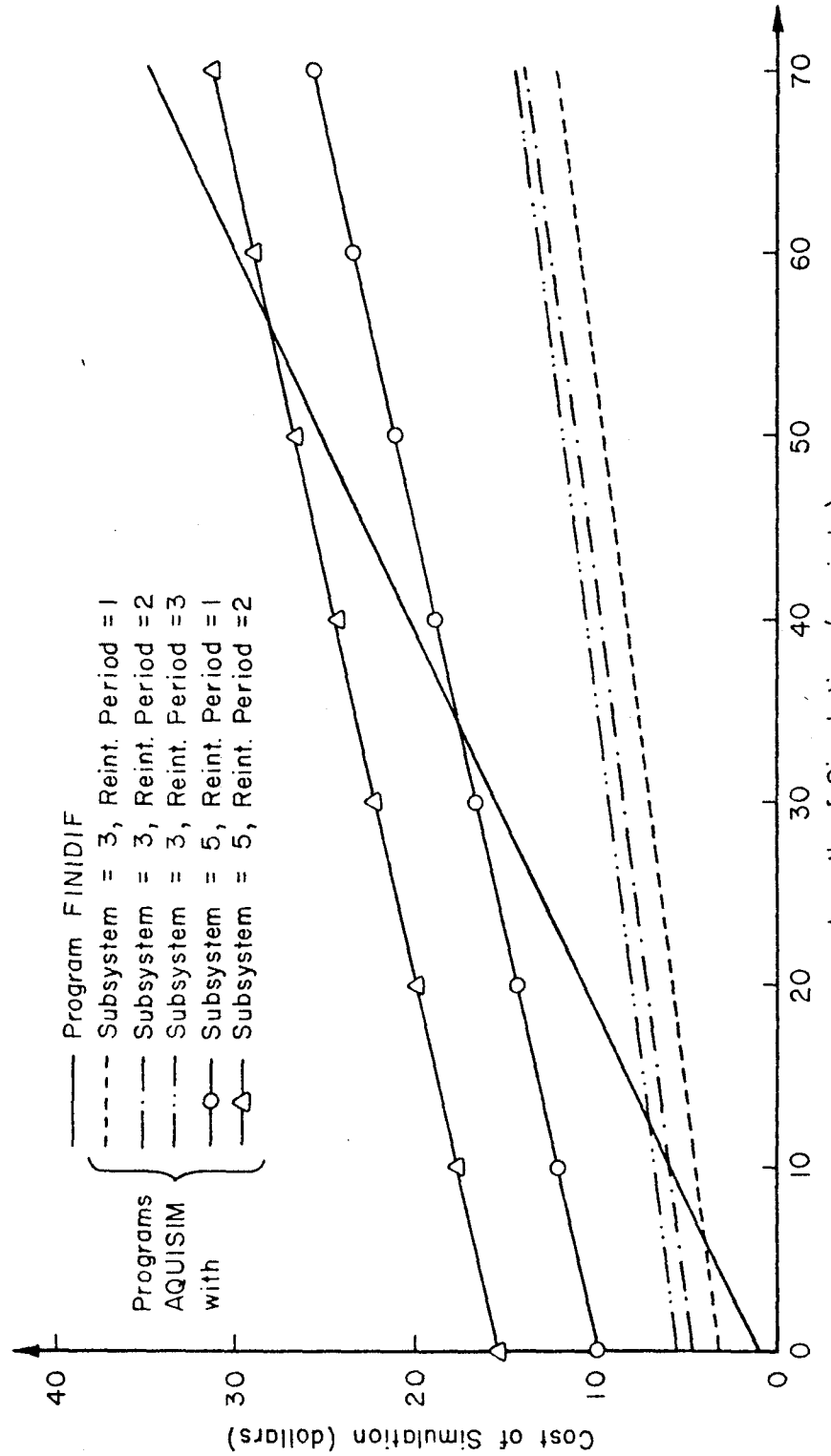

Figure 17. Cost of simulation using FINIDIF and AQUISIM as a function of length of simulation

initial costs due to the required generation of the discrete kernels. With long simulation periods, the real cost of using programs AQUISIM decreases.

The real advantage in programs AQUISIM lies in the fact that the impact of numerous management strategies can be assessed without the costly generation of the coefficients. Once the discrete kernels have been generated and saved, any number of pumping strategies or initial conditions can be modeled using only program SIMULAT. The standard finite difference approach requires the solution of the system of equations for each simulation.

Figure 18 shows this effect graphically. As can be seen, with programs AQUISIM, there is an initial outlay of expenses for the coefficient generation. Once these are saved, however, the cost per simulation run is very minimal. The steep gradient of the line representing the costs associated with program FINIDIF indicates the expense involved in generating and solving the system of equations for each management scheme.

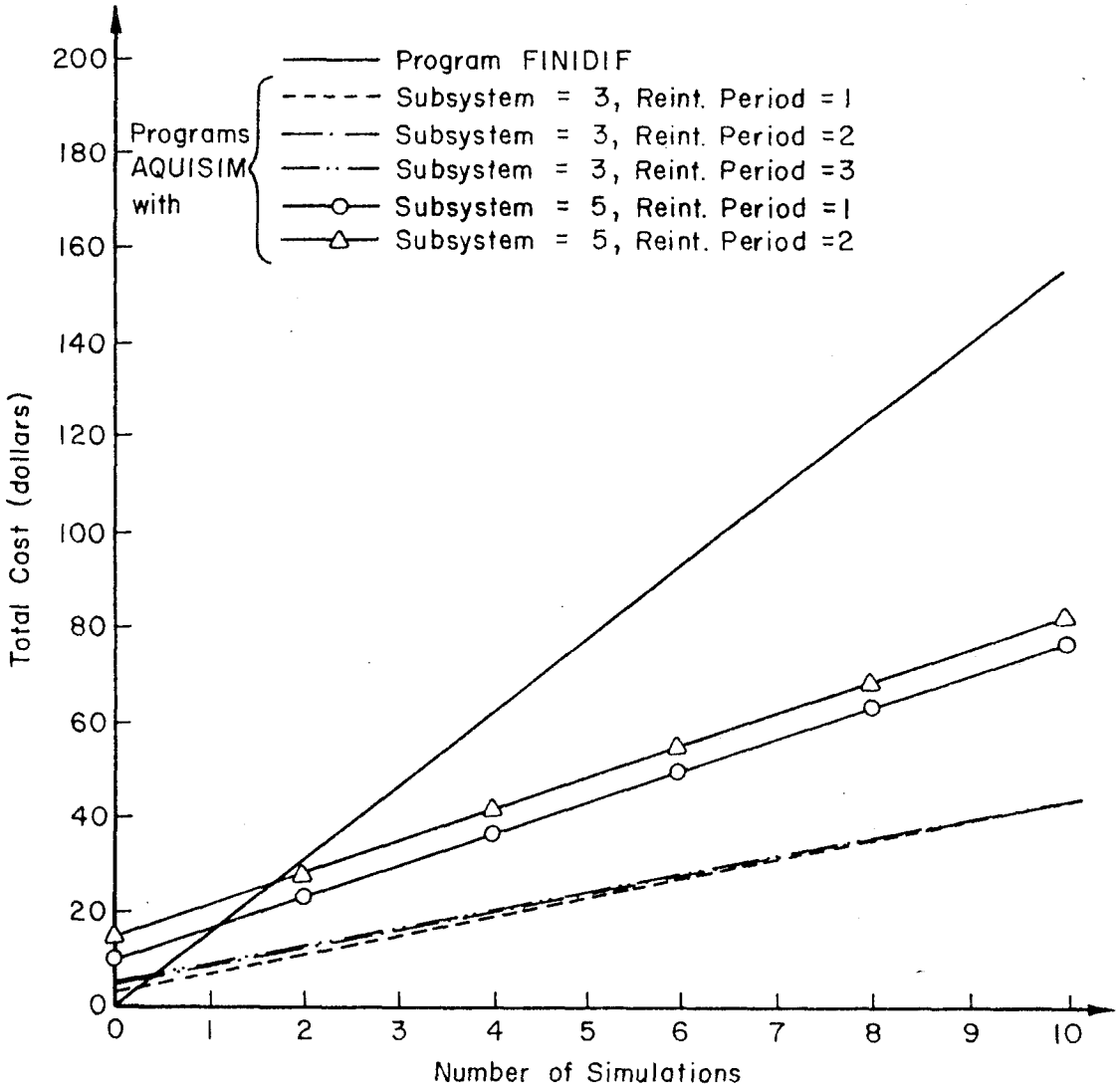

Figure 18. Cost of simulation using FINIDIF and AQUISIM as a function of the number of management strategies

CONCLUSIONS

A two-dimensional numerical-analytical ground water model has been developed which can cost-effectively and accurately portray the response of an unconfined or confined aquifer system to various areal or point excitations. The output produced by this model is at least as accurate as that provided with standard finite difference models. In many cases, the output is closer to the analytic solution than standard models. The study presented indicates that use of this model can reduce the cost involved in using computer models if judicious selection of both subsystem size and reinitialization period is made. The developed model is particularly efficient if more than one simulation run are to be made on the same system.

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