MATHEMATICAL MODELING OF WATER MANAGEMENT STRATEGIES IN URBANIZING RIVER BASINS

by

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ABSTRACT

MATHEMATICAL MODELING OF WATER MANAGEMENT STRATEGIES IN URBANIZING RIVER BASINS

Water management in arid urbanizing regions requires careful evaluation of the alternative strategies for supply-plying future demands and control of water quality. Mathematical models were formulated to study the interrelationships that exist among various institutional factors in order to delineate the requirements for implementing optimal policies. Two study areas were selected on which to test the utility of the models. Denver, Colorado, was chosen as an area representing conditions of water scarcity and increasingly stringent water quality standards. The Utah Lake drainage area in central Utah presented conditions where water quality management is necessary to insure the continued use of water in the downstream population center of the state. Together these models produce results useful in determining the optimal strategies for water management in arid urbanizing areas.

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KEYWORDS - agricultural wastes, institutional constraints, mathematical models, optimization, systems analysis, urbanization, wastewater treatment, water management (applied), water quality, water supply.

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NOMENCLATURE

Symbol	Definition	Units
A_1, A_2, A_3	polynomial regression coefficients . . .	-
B_A, B_o, B_u	BOD concentrations in district model	mg/l
BOD	biochemical oxygen demand	mg/l
b	per capita BOD production	lbs
$\underline{C}$	gradient of the constraints with respect to the decision variables	-
C_k	concentration of TDS and BOD at flow control points in models . . .	mg/l
C_p	total costs of improving practices in the agricultural sector	$
C_s	total costs of structural improvements in agricultural sector . . .	$
D_k	urban demands	mgd
$\underline{d}$	decision variables	-
$\underline{\bar{f}}$	active constraint set	-
$\underline{\bar{H}}$	Hession Matrix	-
$\underline{\bar{J}}$	Jacobian Matrix	-
$\bar{P}_k$	constants used to store system costs	$
PE	population equivalent	-
Q_k	flow rate constants	mgd
$\underline{s}$	state variables	-
$\bar{T}DS$	total dissolved solids	mg/l
T_k	removal efficiencies	-
X_k	model flow rate variables	mgd
$\underline{x}^0$	initial feasible solutions	-
$\underline{x}^\nu$	new feasible solutions	-
$\bar{Y}$	capital construction costs	$
Y_o	operation and maintenance costs	$
y	value of objective function	-
y	operation and maintenance costs	¢/1000 gal
Z	facility capacities	mgd
ϕ	slack variables	-
∇_s	gradient with respect to state variables	-
∇_d	gradient with respect to decision variables	-
δ	constrained derivatives	-
∂	partial derivatives	-

SECTION I

INTRODUCTION

Purpose

An essential requirement for advancing civilizations has been to increase agricultural production. A few centuries ago a single farmer could barely support his family, but modern agriculturalists are capable of supplying food and fibre for many. The evolution of the agricultural enterprise from the individualistic subsistance farming to the corporate business has also reduced the number of people necessary to satisfy agricultural demands. Consequently, with fewer opportunities in the agricultural industry, people have aggregrated in metropolitan environments to work in government administration, support services, and manufacturing to note only a few. A basic shift has thus occurred from rural to urban living.

Regional urbanization has been accompanied by new problems in administering natural resources such as water. First, the demands for water of suitable quality have greatly affected the usefulness of water supplies in several areas. As a result, new sources have been actively sought and the feasibility of employing technological advances to amend marginal supplies have been investigated. Secondly, the concentration of water use in conjunction with the growing demands have created serious water quality degradation by far exceeding the natural assimilative capacity of rivers and lakes. And finally, the institutional mechanisms developed to allocate and manage the water resource have not been altered sufficiently to effectively meet the requirements of rapid urbanization.

The purpose of this study is to investigate the feasibility of alternative water management strategies which could be implemented to alleviate the mounting problems of water shortage and water quality deterioration. At this level of interest, the factors especially requiring evaluation are the institutional requirements for accomplishing efficient operation of water use systems. The objective therefore is to model alternative water management strategies to test the effects of various institutional factors in accomplishing more effective water use.

Scope

Administering water resource utilization in rapidly urbanizing areas encompasses numerous individual aspects of significant importance. Two of these have been selected for this study:

(1) coordination of the supply, distribution, and treatment of water in the metropolitan setting; and

(2) regional integration of agricultural and urban water pollution control.

Since the first topic is an important subproblem of the second, consideration of these two problems allows this study to evaluate the specific institutional requirements for optimizing water management decisions in urbanizing areas.

Two regions where the conditions are particularly suitable to the objectives of this study are the Denver, Colorado metropolitan area and the Utah Lake Drainage area in central Utah.

The Denver, Colorado, area is a water-short, rapidly expanding urban center facing rigid water quality controls. Water supplies for the area are primarily obtained from sources in both the headwaters of the South Platte River Basin and the headwaters of the nearby Colorado River Basin. It appears reasonable to conclude from previous developments, that diverting more water resources from these watersheds to supply the expanding needs will induce extensive legal, social, and political controversy. A need therefore exists to evaluate the feasibility of alternative management strategies in this area and determine the nature and expense of the institutional constraints which may hinder implementation of more effective policies.

The Utah Lake Drainage area does not face serious water supply problems, but it does contribute significantly to a critical water quality problem in its downstream reaches. This region is the headwaters of the Jordan River, which is a major water source for three-fourths of Utah's population located along an area known as the "Wasatch Front." The water demands along the Wasatch Front are comprised mainly of municipal and industrial uses, which require not only a sufficient supply but an acceptable quality as well. Efforts to minimize water pollution must therefore be regional in scope, which necessitates examination of practices and potential treatments in the Utah Lake Drainage area. This study is thus concerned with the levels of water quality control achievable in the area and the optimal policies to accomplish such control.

In order to present the results of this study, four reports have been prepared. This effort is the first of the series and covers the modeling procedures employed in the study. The first segment of this report, encompassing SECTIONS II and III, deal with the mechanics of the optimization processes used to evaluate alternative strategies. Next, the urban water management model is formulated in SECTIONS IV and V, which expresses the scope of the investigation in the Denver metropolitan area. Next, the Utah Lake modeling techniques are illustrated in SECTIONS VI and VII. Finally, SECTION VIII summarizes the modeling efforts and recommends additional research, as well as suggestions for improving these models.

SECTION II

OPTIMIZATION CRITERION

Introduction

Alternative measures for meeting the requirements of water management problems in areas of urbanization need to be evaluated for feasibility in the context of both long and short range objectives. In order to facilitate such comparisons necessitates a criterion upon which a common link between alternatives can be developed. This Section presents some general comment and support from other investigators for the optimization criterion selected for this study.

Economic Nature of Water Resource Systems

There is probably no other means as commonly used or as widely accepted for evaluating the merits of water resource systems as is economics. While environmental concerns have been mounting and engineering designs have become more sophisticated, the central character in evaluating projects is the economic analysis. Not all of these economic considerations have been made by economists, but those making the studies have of necessity relied upon the discipline to provide new and better techniques for in-investigation.

Although water resources can be classified primarily as public commodities, significant influences on pricing and management are due to water uses in the private market. In most states, water is not legally "owned" by an individual other than the state, but rights can be obtained for the use of water by individuals. However, when the legal interpretation implies that the water is tied to the land and cannot be transferred, then the value of the land is enhanced by its water right. These cases give water a market value obtainable by a right holder even when the resource is administered as public property. As in the case of grazing privileges on public lands, the pricing is usually lower than that obtainable in the private economy. As a consequence, right holders are often reluctant to accept changes which may reduce their water supply.

Reservoirs, diversion works, and distribution systems aid management of water resources which tend to remain fixed in spatial distribution and random in time distribution. These characteristics which would otherwise constrain water supplies to local utilization, allow wider water use between adjoining watersheds and along a river system. However, the diversion of waters from one basin to another, or the transfer of water usage to another location in the river network, creates exteralities which are usually not considered by local planners. Thus, maximum economic efficiencies are only achieved when the economic evaluations assume a regional interpretation.

Finally, the inefficiencies existing in current water use practices can be traced to a large extent to those social, legal, and political institutions responsible for distribution of water among demands. These limitations have not been severe until water resources have become scarce. However, when expansions in urban needs occur, the water resources that could be better utilized in a new use may be tied to an old use without means for making a conversion. As a result, the optimal water management policies which suggest that water be transferred from one use to another, such as transferring agricultural water to municipal uses, have been difficult to date because of the institutional constraints (Hartman and Seastone, 1972). Although such constraints hinder efficient use of water, they have nevertheless become the tools which substitute for the free market economic system.

Optimizing Criterion

Optimization is generally a maximization or a minimization of concise numerical quantities reflecting the relative importance of the goals and purposes contained in alternative decisions. Of themselves, neither the goals or purposes directly yield the precise quantitative statements required by systems analysis procedures. Therefore, the objectives to be accomplished must first be stated by a quantitative measure from which alternative policies can be mathematically compared (Hall and Dracup, 1970).

Presumably, such a comparison would permit a ranking of these policies as a basis for decision making. The specific measure to facilitate this examination can be defined as the optimizing criterion.

The central problem facing engineers is to link the descriptions of the physical environment via mathematical models with the social and political environment (Thomann, 1972). Probably the most commonly used and widely accepted "indicators" are found among the many economic objective functions. However, considerable controversy exists as to the most realistic of these tools. If all human desires could be priced in an idealized free market monetary exchange, the forces that operated would insure that every individual's marginal costs equalled his marginal gains, thereby insuring maximum economic efficiency. In fact, such a condition would reduce the need for optimization methodologies to aid decision making. In the absence of this ideal situation, goals cannot be quantified with a high degree of accuracy and the optimizing criterion in any case is at best an indicator of the particular alternative.

Among the more adaptable economic indicators are maximization of net benefits, minimum costs, maintaining the economy, and economic development. The use of each depends on the ability to adequately define tangible and intangible direct or indirect costs and benefits. In water resource development and water quality management specifically, the economic incentives for more effective resource utilization are negative in nature (Kneese, 1964). A large part of this problem stems from the fact that water pollution is a cost passed on by the polluter to the downstream user. Consequently, the inability of the existing economic systems to adequately value costs and benefits has resulted in the establishment of water quality standards, however inefficient these may be economically (Hall and Dracup, 1970). The immediate objective of water resource planners is thus to devise and analyze the alternatives for achieving these quality restrictions at minimum cost (Thomann, 1972) and is the criteria chosen for this study.

SECTION III

JACOBIAN DIFFERENTIAL ALGORITHM

Introduction

The search for an optimizing technique to evaluate the relative merits of an array of alternatives depends largely upon the form of the problem and its constraints. While the allegorical Chinese maxim cited by Wilde and Beightler (1967) stating, "There are many paths to the top of the mountain, but the view there is always the same," is also true in this case; not every method can be applied with the same ease. Each optimization scheme has its unique properties making it adaptable to specific problems, although many techniques when sufficiently understood can be modified to extend their applicability. Successful modifications of this nature are prevalent in current engineering practice but requires some experience in using these methods.

Most conditions encountered in the field of water resources, urban water systems specifically, involve mathematical formulations which are non-linear in both the objective function and the constraints. Furthermore, the constraining functions may be mixtures of linear and non-linear equalities and inequalities. Without simplifying these problems or radically changing existing optimization techniques, it is possible to derive solutions based upon what Wilde and Beightler (1967) describe as the "differential appraoch."

Most techniques for selecting the optimal policy do so by successively improving a previous estimate until no betterment is possible. These may be classified as direct or indirect methods depending on whether they start at a feasible point and stepwise move toward the optimum or solve a set of equations which contain the optimum as a root. In a majority of cases, the differential approach can be used to describe the method. Thus, it is possible to understand a wide variety of procedures by knowing one basic mathematical approach.

Numerous applications of one form or another of the basic differential approach have been made in the field of engineering (Monarchi, 1972). Because of the considerable difficulty in programming "generality," nearly all of these applications have been somewhat specialized toward the specific geometry of the problem. The research project responsible for this development necessitates two entirely different optimization analyses. Consequently, to avoid developing two models, it was decided to attempt to program a general differential algorithm. A class entitled "Foundations of Engineering Optimization" taught by Dr. H. J. Morel-Seytoux, Professor of Civil Engineering at Colorado State University provided the theoretical basis for the model. This writer, a student in the class, coded the algorithm for use on the digital computer facilities at the University.

The optimizing technique is called in this writing the "Jacobian Differential Algorithm." Theoretically, it is a generalized eliminating procedure which is computationally feasible under a wide variety of conditions. The characteristics of convexity are assumed and since the maximization problem is simply the negative of a minimization one, the succeeding discussion will be limited to the latter case. As in all direct minimizing procedures, the algorithm involves four steps:

1. Evaluate a first feasible solution, $\underline{x}^o$ which satisfies the problem constraints. The underbar indicates vector notation and the superscript ^{o} is used to describe the "old" or initial points.
2. Determine the direction in which to move such that the objective function, y, is decreased the most rapidly. This requires a move from $\underline{x}^o$ to the new point, $\underline{x}^\nu$ in which the superscript $^\nu$ represents the new point notation.
3. Find the distance that can be moved without violating any of the problem constraints.
4. Stop when the optimum is reached.

While the procedure yields the requirements for steps 2 and 3, the user is left with providing the first feasible solution, step 1. This may seem to be a drawback for the problem, but in real situations a feasible solution already exists as a current policy. Step 4 is accomplished by an examination of what are now referred to as the "Kuhn-Tucker conditions." These criteria do not indicate whether the procedure has reached a local or global optimum; consequently, it is necessary to derive a means for checking. This is not usually a difficult process.

Theoretical Development

Consider the problem in which the minimum value of the objective function is sought subject to a set of constraining functions. Writing this problem mathematically,

$$\min_{x} \{y = y(\underline{x})\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

subject to,

$$\underline{f}(\underline{x}) \geq \underline{0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where the notation $y(\underline{x})$ denotes "as a function of the vector x." The number of x variables is defined as N and the number of constraints as K. The method of analysis depends largely upon the structure of the constraints. When all the constraints are inequalities and "loose" or "inactive" (strictly >) at the initial feasible point $\underline{x}^o$, the problem is "unconstrained." In the other case when either some of these functions are strict equalities or when some of the inequalities are "tight" or "active," the problem is referred to as "constrained." Although both of the conditions may occur in the solution of a problem, they require somewhat different approaches as the algorithm progresses toward the optimum.

Elimination Procedure

The elimination nature of the technique is derived from the fact that it is at least conceptually possible to employ only the currently active constraints to eliminate some of the x's from the problem, making it temporarily unconstrained. To begin, define the number of active constraints as T and reorder the constraint set so that the first T are the active constraints with index t = 1, 2, . . ., T. Further, introduce "slack" variables to the active constraints so they take the form,

$$\underline{f}(\underline{x}) - \underline{\phi} = \underline{0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

and become strict equalities, where $\underline{\phi}$ is the vector of slack variables. Later in the development, slack variables will also be added to the inactive constraints. The purpose of this transformation is that by continual observation of the slack values, the distinction between active and inactive functions can be determined, since active slack variables are equal to zero and inactive slacks are always greater than zero. The problem now contains N original variables plus T slack variables which are related by T active constraints. If the constraints are linear, T of the variables can be eliminated from the objective function by the constraint expressions, making the problem unconstrained. However, in the general situation, the constraints are nonlinear, and it is not directly possible to substitute for the dependent variables. It is necessary in the general case to first linearize the functions by taking the first partial derivatives with respect to the x variables. Even though the non-linearity may still exist due to the nature of the terms in the constraints, if it is assumed that the changes toward the optimum point are sufficiently small, then only a small deviation is introduced. The elimination procedure takes place by partitioning the variable set into "states" and "decisions." The state variables are the selected variables which are to be eliminated by the T active constraints. The decision variables are the remaining independent variables which will be employed to seek the minimum value of the objective function. The criteria for the partition include two aspects:

1. All slack variables are taken as decisions unless no other x-variable is available to be a state variable. Since all ϕ_t are identically equal to zero, when the algorithm moves from the old point $\underline{x}^o$ to the new one $\underline{x}^\nu$ in its search for the minimum, there is a 50 percent chance that the ϕ_t will become negative. This is a violation of the problem constraints.
2. Since the same basic reasoning applies to the x-variables, the largest absolute valued variables are best suited to be state variables.

In the computer code of the algorithm, the selection of states and decisions is much more complex, but to describe all the partitioning difficulties at this point would be confusing.

After partitioning the x-vector into state and decision variables, the variables can be relabeled s for states and d for decisions. Equation 1 at the initial point $\underline{x}^o$ can then be written,

$$\min_{\underline{d}} \quad y = y(s_1, s_2, \ldots, s_T, d_1, d_2, \ldots, d_D) \quad . \quad . \quad (4)$$

in which D is the number of decision variables and equals (N - T). In addition, the constraints listed in Equation 3 can be rewritten as:

$$\underline{f}(\underline{s},\underline{d}) - \underline{\phi} = \underline{0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The next step is to employ the chain rule of calculating the total differential of y. In vector notation,

$$\partial y = (\nabla_s y)\partial \underline{s} + (\nabla_d y)\partial \underline{d} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where the symbol ∂y is used to denote the total differential rather than the standard notation of dy. This modification is made so that the d can be reserved to denote the decision variables.

The derivatives of the constraining functions can also be written in vector form,

$$(\nabla_s \underline{f})\partial \underline{s} + (\nabla_d \underline{f})\partial \underline{d} - \partial \underline{\phi} = \underline{0} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where the gradient, $(\nabla_s \underline{f})$, is called the Jacobian Matrix, $\underline{J}$, and the matrix $(\nabla_d \underline{f})$ can be relabeled as $\underline{C}$. Employing these variables in Equation 7 and rearranging terms:

$$\underline{J}\partial \underline{s} = -\underline{C}\partial \underline{d} + \partial \underline{\phi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

If the Jacobian matrix is always taken non-singular, the vector $\partial \underline{s}$ can be solved for.

$$\partial \underline{s} = -\underline{J}^{-1}\underline{C}\partial \underline{d} + \underline{J}^{-1}\partial \underline{\phi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

The elimination of the states is now possible by substitution of Equation 9 into Equation 6. After rearranging terms, the final unconstrained equation is developed.

$$\partial y = \left[\nabla_d y - (\nabla_s y)\underline{J}^{-1}\underline{C}\right] \partial \underline{d} + (\nabla_s y)\underline{J}^{-1}\partial \underline{\phi} \quad . \quad . \qquad (10)$$

Kuhn-Tucker Conditions

At this point, the key parameters in the Jacobian Differential Algorithm can be introduced. By definition of the total differential, another expression can be written in terms of the variables indicated in Equation 10. If the elimination of the state differentials was accomplished then the total differential of y would be written,

$$\partial y = \frac{\delta y}{\delta \underline{d}} \partial \underline{d} + \frac{\delta y}{\delta \underline{\phi}} \partial \underline{\phi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (11)$$

in which $\delta y/\delta \underline{d}$ and $\delta y/\delta \underline{\phi}$ are called "constrained derivatives." The deviation in notation is made to distinguish the $\partial y/\partial \underline{x}$, which is a partial derivative viewing all variables as independent, from $\delta y/\delta \underline{d}$ which is a partial derivative considering T of the variables as functions of the remaining N variables. By comparing Equations 10 and 11 it can be seen that,

$$\frac{\delta y}{\delta \underline{d}} = \nabla_d y - (\nabla_s y)\underline{J}^{-1}\underline{C} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (12)$$

and,

$$\frac{\delta y}{\delta \underline{\phi}} = (\nabla_s y)\underline{J}^{-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (13)$$

The solution of Equations 12 and 13 when equated to zero yield a stationarity point when the decision variables are free, or in other words, allowed to assume any positive or negative value. In most instances, decision variables are not free, but subject to non-negativity conditions. Stationarity points may be local or global minimums, maximums, or inflection points. The evaluation of stationarity points in these cases will depend on criteria reported by Kuhn and Tucker (1951) which provide necessary and sufficient conditions for a minimum. In the problem solution at the feasible point under examination, a minimum exists if the following conditions are met:

1. Necessary conditions prerequisite for a minimum must consist of the following:

$$\frac{\delta y}{\delta d_j} \geq 0,\ d_j \geq 0,\ \text{and}\ \frac{\delta y}{\delta d_j} d_j = 0 \quad j = 1,2,\ldots,D \qquad (14)$$

and

$$\frac{\delta y}{\delta \phi_t} \geq 0,\ \phi_t \geq 0,\ \text{and}\ \frac{\delta y}{\delta \phi_t} \phi_t = 0,\ t = 1,2,\ldots,T \qquad (15)$$

2. If Equations 14 and 15 are satisfied, then sufficient conditions for a minimum are:

$$\frac{\delta y}{\delta d_j} > 0 \quad j = 1,2,\ldots,D \quad . \quad . \quad . \quad . \qquad (16)$$

and

$$\frac{\delta d}{\delta \phi_t} > 0 \quad t = 1,2,\ldots,T \quad . \quad . \quad . \quad . \qquad (17)$$

The minimum has been reached when both the necessary and sufficient conditions have been satisfied. However, if for example, $\delta y/\delta d_j$ equals zero and $d_j \geq 0$, the tests are inconclusive since the sufficient conditions have not been met. In this case, it is necessary to take the second derivatives of the objective function with respect to the x-vector. This analysis yields a square matrix of second order partial derivatives called the Hessian matrix written mathematically as:

$$\underline{H} = \nabla_x^2 y \quad . \quad . \quad . \quad . \quad . \quad . \qquad (18)$$

In order for the stationarity point to be a minimum (local or global) the value of the Hessian matrix must be positive-definite, and since the properties of positive-definite matrices can be found in most texts on linear algebra, no further description will be given here.

Evaluation of Optimal Direction

In addition to the description of the fundamental elimination technique of this optimizing technique, the preceding sections also provided the definition of the constrained derivatives of the objective function in terms of the decision and slack variables. Furthermore, criteria were given with which these parameters can also be evaluated to see when the minimum is achieved. In this section, these same derivatives will be used to determine the direction a particular decision variable, d_p or ϕ_p, must be "moved" in order to create the maximum reduction in the value of the objective function during each iterative step.

Among the non-linear programming techniques for optimization several essentially alter all of the decision variables at each iteration. In the Jacobian Differential Algorithm, one decision variable (d_p or ϕ_p) is selected from among the set which when moved will result in the most progress toward the minimum. If an individual term from Equation 11 is written in discrete element form, the new value of the decision variable (or slack variable) can be determined,

$$y^{\nu} - y^{o} = \left(\frac{\delta y}{\delta d_i}\right)^{o} \left(d_i^{\nu} - d_i^{o}\right) \quad . \quad . \quad . \quad . \quad (19)$$

or,

$$y^{\nu} - y^{o} = \left(\frac{\delta y}{\delta \phi_t}\right)^{o} \phi_t^{\nu} \quad . \quad . \quad . \quad . \quad . \quad (20)$$

where the reader is reminded that the superscripts ^{o} and $^{\nu}$ refer to the functional evaluations made at the old and new feasible solutions. It may also be worth mentioning that ϕ_t can only be increased whereas d_i can be also decreased (assuming the non-negativity constraints are not violated). As a result, the increase in a slack variable is in reality a loosening of an active constraint.

The choice of the decision variable or the slack variable to be modified is primarily made on the basis of largest absolute value among the respective constrained derivatives. Three general categories are examined. To begin with, the largest positive valued derivative with which the associated decision variable is greater than zero is determined and the Kuhn-Tucker Conditions are checked according to the previous section. Mathematically, this first alternative can be written,

$$\text{find:} \quad \max_i \left[\frac{\delta y}{\delta d_i} > 0 \quad \middle| d_i > 0,\ i = 1,2,\ldots,D \right] \qquad (21)$$

where the notation $|d_i > 0$ means "subject to the value of d_i being positive."

The second alternative selection for the step direction is in the negative constrained derivatives. In this case, the specific decision variable will be increased and unless an upper bound on the variable is imposed, no examination of the decision need be made. Symbolically then,

$$\text{find:} \quad \min_i \left[\frac{\delta y}{\delta d_i} < 0,\ i = 1,2,\ldots, D \right] . \quad . \quad . \ (22)$$

Finally, the largest reduction in the objective function may be facilitated by loosening a particular active constraint. Unless the constrained derivative of y with respect to the slack variable is negative, the Kuhn-Tucker Conditions are satisfied. Therefore, this solution can be expressed as:

$$\text{find:} \quad \min_t \left[\frac{\delta y}{\delta \phi_t} > 0,\ t = 1,2,\ldots, T \right] \quad . \quad . \ (23)$$

Once these maximum and minimums have been selected, the next item is to compare them with each other and select the largest absolute valued one. After having made the choice, the index on the specified decision or slack variable is now denoted by a "p", and these variables now become d_p or ϕ_p depending on the decision among alternatives.

Determining the Step Size

The analysis in the previous section paved the way to compute the direction in which the decision and slack variables are to be moved for a maximum decrease in the objective function. This section is presented to find how much the procedure can move in the appropriate direction without violating the constraints. In order to accomplish this, four new constrained derivatives must be developed. To begin, it is useful to rewrite Equations 6 and 7 as a complete differential system. In addition, the slack variables (ϕ^+) may be added to the inactive constraints (f^+) and included in the differential system. The complete three-part system has been included in Figure 1.

Because the particular decision variable or slack variable to be modified has been selected, the remaining decisions and slacks will remain constant and can therefore be temporarily ignored. The next computation necessary is to determine which of the boundaries of the problem are approached first. If the non-negativity constraints on the variables are in effect, one consideration is how far a decision or slack variable can be moved without forcing a state variable to become negative. In order to accomplish this, the constrained derivatives of each state variable with respect to the particular decision or slack variable are computed. The representation of these values is computed from the formulas shown in Figure 2 in which the use of Cramer's rule was applied to the system in Figure 1.

$$-\partial y + \frac{\partial y}{\partial s_1}\partial s_1 + \frac{\partial y}{\partial s_2}\partial s_2 + \dots + \frac{\partial y}{\partial s_T}\partial s_T = -\frac{\partial y}{\partial d_1}\partial d_1 - \frac{\partial y}{\partial d_2}\partial d_2 - \dots - \frac{\partial y}{\partial d_D}\partial d_D$$

$$\frac{\partial f_1}{\partial s_1}\partial s_1 + \frac{\partial f_1}{\partial s_2}\partial s_2 + \dots + \frac{\partial f_1}{\partial s_T}\partial s_T = -\frac{\partial f_1}{\partial d_1}\partial d_1 - \frac{\partial f_1}{\partial d_2}\partial d_2 - \dots - \frac{\partial f_1}{\partial d_D}\partial d_D + \partial \phi_1$$

$$\vdots$$

$$\frac{\partial f_t}{\partial s_1}\partial s_1 + \frac{\partial f_t}{\partial s_2}\partial s_2 + \dots + \frac{\partial f_t}{\partial s_T}\partial s_T = -\frac{\partial f_t}{\partial d_1}\partial d_1 - \frac{\partial f_t}{\partial d_2}\partial d_2 - \dots - \frac{\partial f_t}{\partial d_D}\partial d_D + \partial \phi_t$$

$$\vdots$$

$$\frac{\partial f_T}{\partial s_1}\partial s_1 + \frac{\partial f_T}{\partial s_2}\partial s_2 + \dots + \frac{\partial f_T}{\partial s_T}\partial s_T = -\frac{\partial f_T}{\partial d_1}\partial d_1 - \frac{\partial f_T}{\partial d_2}\partial d_2 - \dots - \frac{\partial f_T}{\partial d_D}\partial d_D + \partial \phi_T$$

$$-\partial \phi_1^+ + \frac{\partial f_1^+}{\partial s_1}\partial s_1 + \frac{\partial f_1^+}{\partial s_2}\partial s_2 + \dots + \frac{\partial f_1^+}{\partial s_T}\partial s_T = -\frac{\partial f_1^+}{\partial d_1}\partial d_1 - \frac{\partial f_1^+}{\partial d_2}\partial d_2 - \dots - \frac{\partial f_1^+}{\partial d_D}\partial d_D$$

$$\vdots$$

$$-\partial \phi_\ell^+ + \frac{\partial f_\ell^+}{\partial s_1}\partial s_1 + \frac{\partial f_\ell^+}{\partial s_2}\partial s_2 + \dots + \frac{\partial f_\ell^+}{\partial s_T}\partial s_T = -\frac{\partial f_\ell^+}{\partial d_1}\partial d_1 - \frac{\partial f_\ell^+}{\partial d_2}\partial d_2 - \dots - \frac{\partial f_\ell^+}{\partial d_D}\partial d_D$$

$$\vdots$$

$$-\partial \phi_L^+ + \frac{\partial f_L^+}{\partial s_1}\partial s_1 + \frac{\partial f_L^+}{\partial s_2}\partial s_2 + \dots + \frac{\partial f_L^+}{\partial s_T}\partial s_T = -\frac{\partial f_L^+}{\partial d_1}\partial d_1 - \frac{\partial f_L^+}{\partial d_2}\partial d_2 - \dots - \frac{\partial f_L^+}{\partial d_D}\partial d_D$$

Figure 1. Differential system expressing the linearized objective function, active constraints, and inactive constraints.

$$\frac{\delta s_i}{\delta d_p} = - \frac{\begin{vmatrix} \frac{\partial f_1}{\partial s_1} & \cdots & \frac{\partial f_1}{\partial s_{i-1}} & \frac{\partial f_1}{\partial d_p} & \frac{\partial f_1}{\partial s_{i+1}} & \cdots & \frac{\partial f_1}{\partial s_T} \\ \vdots & & & & & & \\ \frac{\partial f_t}{\partial s_1} & \cdots & \frac{\partial f_t}{\partial s_{i-1}} & \frac{\partial f_t}{\partial d_p} & \frac{\partial f_t}{\partial s_{i+1}} & \cdots & \frac{\partial f_t}{\partial s_T} \\ \vdots & & & & & & \\ \frac{\partial f_T}{\partial s_1} & \cdots & \frac{\partial f_T}{\partial s_{i-1}} & \frac{\partial f_T}{\partial d_p} & \frac{\partial f_T}{\partial s_{i+1}} & \cdots & \frac{\partial f_T}{\partial s_T} \end{vmatrix}}{|\underline{J}|}$$

$$\frac{\delta s_i}{\delta \phi_p} = (-1)^{t+i} \frac{\begin{vmatrix} \frac{\partial f_1}{\partial s_1} & \cdots & \frac{\partial f_1}{\partial s_{i-1}} & \frac{\partial f_1}{\partial s_{i+1}} & \cdots & \frac{\partial f_1}{\partial s_T} \\ \vdots & & & & & \\ \frac{\partial f_{t-1}}{\partial s_1} & \cdots & \frac{\partial f_{t-1}}{\partial s_{i-1}} & \frac{\partial f_{t-1}}{\partial s_{i+1}} & \cdots & \frac{\partial f_{t-1}}{\partial s_T} \\ \frac{\partial f_{t+1}}{\partial s_1} & \cdots & \frac{\partial f_{t+1}}{\partial s_{i-1}} & \frac{\partial f_{t+1}}{\partial s_{i+1}} & \cdots & \frac{\partial f_{t+1}}{\partial s_T} \\ \vdots & & & & & \\ \frac{\partial f_T}{\partial s_1} & \cdots & \frac{\partial f_T}{\partial s_{i-1}} & \frac{\partial f_T}{\partial s_{i+1}} & \cdots & \frac{\partial f_T}{\partial s_T} \end{vmatrix}}{|\underline{J}|}$$

Figure 2. Algebraic formulas for computing the state variable constrained derivatives with respect to the particular decision or slack variables.

From these values, the maximum move may be computed. Writing the appropriate relationships in discrete form,

$$(s_i^{\nu} - s_i^{o}) = \left(\frac{\delta s_i}{\delta d_p}\right)^{o} \left(d_p^{\nu} - d_p^{o}\right) \quad . \quad . \quad . \quad . \quad (24)$$

or for the slack variables:

$$(s_i^{\nu} - s_i^{o}) = \left(\frac{\delta s_i}{\delta \phi_p}\right)^{o} \quad \phi_p^{\nu} \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

Three cases exist in which a state variable can be driven to zero, namely a decrease in d_p, an increase in d_p, and an increase (or loosening) in ϕ_p. Since a search is necessary among the state variables to see which specific state goes to zero first, Equations 24 and 25 can be incorporated:

Case 1. Decreasing d_p

$$d_p^{\nu} = \max_i \left[d_p^{o} - \frac{s_i^{o}}{\left(\frac{\delta s_i}{\delta d_p}\right)^{o}} \;\middle|\; \frac{\delta s_i}{\delta d_p} > 0 \right] \quad . \quad . \quad (26)$$

Case 2. Increasing d_p

$$d_p^{\nu} = \min_i \left[d_p^{o} - \frac{s_i^{o}}{\left(\frac{\delta s_i}{\delta d_p}\right)^{o}} \;\middle|\; \frac{\delta s_i}{\delta d_p} < 0 \right] \quad . \quad . \quad (27)$$

Case 3. Increasing ϕ_p.

$$\phi_p^{\nu} = \min_i \left[- \frac{s_i^{o}}{\left(\frac{\delta s_i}{\delta \phi_p}\right)^{o}} \;\middle|\; \frac{\delta s_i}{\delta \phi_p} < 0 \right] \quad . \quad . \quad (28)$$

The next possible limitation on the change in the decision or slack variables is the forcing of a previously inactive constrant into an active role in the problem. In order to facilitate this analysis, the constrained derivatives of the loose slack variables with respect to the decision and slack variables is computed. A formula for these computations is given in Figure 3. Again, three conditions must be considered:

Case 1. Decreasing d_p

$$d_p^{\nu} = \max_{\ell} \left[d_p^{o} - \frac{(\phi_\ell^+)^o}{\left(\frac{\delta f_\ell^+}{\delta d_p}\right)^o} \;\middle|\; \frac{\delta f_\ell^+}{\delta d_p} > 0 \right] \quad . \quad . \quad (29)$$

Case 2. Increasing d_p

$$d_p^{\nu} = \min_{\ell} \left[d_p^{o} - \frac{(\phi_\ell^+)^o}{\left(\frac{\delta f_\ell^+}{\delta d_p}\right)^o} \;\middle|\; \frac{\delta f_\ell^+}{\delta d_p} < 0 \right] \quad . \quad . \quad (30)$$

Case 3. Increasing ϕ_p

$$\phi_p^{\nu} = \min_{\ell} \left[- \frac{(\phi_\ell^+)^o}{\left(\frac{\delta f_\ell^+}{\delta \phi_p}\right)^o} \;\middle|\; \frac{\delta f_\ell^+}{\delta \phi_p} < 0 \right] \quad . \quad . \quad (31)$$

A final limitation which should be noted is when a decrease in d_p is to be made and neither condition above is violated before non-negativity is encountered. In such a case, the maximum decrease would be $- d_p$ assuming the non-negativity conditions hold. Once this and the other values of d_p and ϕ_p have been made, the most limiting case is evaluated as the proper change in d_p or ϕ_p, whichever the case may be.

$$\frac{\delta f_\ell^+}{\delta d_p} = \frac{\begin{vmatrix} \frac{\partial f_\ell^+}{\partial d_p} & \frac{\partial f_\ell^+}{\partial s_1} & \cdots & \frac{\partial f_\ell^+}{\partial s_T} \\ \frac{\partial f_1}{\partial d_p} & \frac{\partial f_1}{\partial s_1} & \cdots & \frac{\partial f_1}{\partial s_T} \\ \vdots & & & \\ \frac{\partial f_t}{\partial d_p} & \frac{\partial f_t}{\partial s_1} & \cdots & \frac{\partial f_t}{\partial s_T} \\ \vdots & & & \\ \frac{\partial f_T}{\partial d_p} & \frac{\partial f_T}{\partial s_1} & \cdots & \frac{\partial f_T}{\partial s_T} \end{vmatrix}}{|\underline{J}|}$$

$$\frac{\partial f_\ell^+}{\partial \phi_p} = (-1)^{p+1} \frac{\begin{vmatrix} \frac{\partial f_\ell^+}{\partial s_1} & \cdots\cdots & \frac{\partial f_\ell^+}{\partial s_T} \\ \frac{\partial f_1}{\partial s_1} & \cdots\cdots & \frac{\partial f_1}{\partial s_T} \\ \vdots & & \\ \frac{\partial f_{p-1}}{\partial s_1} & \cdots\cdots & \frac{\partial f_{p-1}}{\partial s_T} \\ \frac{\partial f_{p+1}}{\partial s_1} & \cdots\cdots & \frac{\partial f_{p+1}}{\partial s_T} \\ \vdots & & \\ \frac{\partial f_T}{\partial s_1} & \cdots\cdots & \frac{\partial f_T}{\partial s_T} \end{vmatrix}}{|\underline{J}|}$$

Figure 3. Formulas for calculation of the loose slack variable constrained derivatives with respect to the particular decision or slack variables.

Before this section is concluded, a few notes should be made. The first of these is that the number of state variables depends only on the number of active constraints. If by varying a slack or decision variable, a state is driven to zero, a decision variable must be selected to trade positions with the state because of the rick of zero valued state variables. The second point to make is that when a loose constraint is tightened, a new state variable must be selected from the rest of the decision variables. The exception to this is when a loose constraint is tightened by loosening a currently active constraint. In any event, there are so many functions and variables to keep track of, and so many possible alternatives to consider, that the most difficult aspect of this algorithm is the "bookkeeping" that is necessary. This will be demonstrated in the discussion of the computer code.

The Computer Code

Although the theory encompassing this optimization technique is a very powerful one, the computer code of the method has certain inherent limitations. This is not a fault of this particular program, but rather a characteristic of nearly all programs with any degree of sophistication. The utility of any optimum seeking procedure in engineering applications is largely dependent on the economy of use and its generality. It is primarily the latter aspect that limits the subsequent use by an individual unfamilar with the mechanics of the programs' operation. Very few large computer programs are general enough to be used with little or no knowledge of their structure and weak points. The computer code developed in this section is not among these very few, but a great deal of time and effort has been spent in maximizing the generality of the program.

One of the most efficient uses of coding technology is to provide the means whereby segments of programs can be easily modified and used successively for other purposes. In order to facilitate future use of this program, each functional element in the procedure has been identified in a subroutine format. This type of program structure has several important advantages including the ease in which the program can be debugged. In addition, whatever modifications become desirable can be made within the framework of the subroutine without detailed consideration to the remainder of the program. Another advantageous characteristic of the program is that most of the variables are placed in a common storage, thereby making their values accessable from throughout the program.

The Jacobian Differential Algorithm consists of 25 subroutines which have been defined in Table 1. The entire system can be subdivided into seven groups according to their role in the optimizing technique:

1. Problem definition is accomplished in

Table 1. Definition of subroutine functions.

Subroutine	Function
ANSOUT	Output of the optimal solution
ARRAY	Determination of initial variable partition
CONDER	Updates values contained in program storage arrays
DATAOUT	Output of input data and control variables
DECDJ	Decreases the value of a decision variable
DFDX	Derivatives of the constraints, $\partial f/\partial x$
DIFALGO	Coordination of the complete algorithm
DYDX	Derivatives of the objective function, $\partial y/\partial x$
ENDCHEK	Checks problem to insure the search remains in a feasible region
FKOFX	Constraints
GAUSS	Gaussian elimination procedure for solving system of linear equations
INCDJ	Increases the value of a decision variable
INCFT	Loosens a previously active constraint
JACOBI	Computation of the determinant of the Jacobian matrix
JORK	Selection of the decision or slack variable resulting in the most decrease in the value of the objective function
KODFLDD	Constrained derivative, $\delta\phi_\ell{}^+/\delta d_p$
KODFLDF	Constrained derivative, $\delta\phi_\ell{}^+/\delta\phi_p$
KODSDD	Constrained derivative, $\delta s_i/\delta d_p$
KODSDF	Constrained derivative, $\delta s_i/\delta\phi_p$
KODYDD	Constrained derivative, $\delta y/\delta d_j$
KODYDF	Constrained derivative, $\delta y/\delta\phi_j$
KUNTUK	Checks Kuhn-Tucker conditions for a minimum
MPREG	Utility routine for polynomial regressions
NEWTSIM	Newton-Raphson method for solving systems of non-linear equations
YOFX	Computes the value of the objective function

subroutines YOFX, FKOFX, DYDX, and DFDX.

2. Input-Output is provided by the subroutines DATAOUT and ANSOUT.
3. The coordination of the entire program procedure is handled in subroutine DIFALGO.
4. Organization functions in the program are completed in subroutines REORGA and ARRAY.
5. Special computational subroutines include JORK, JACOB1, ENDCHEK, CONDER, KUNTUK, NEWTSIM, and GAUSS.
6. The principal parts of the program are encompassed in subroutines DECDJ, INCDJ, and INCFT which accomplish the step-by-step movement toward the optimum.
7. The calculation of the constrained derivatives is done in the subroutines, KODYDD, KODYDF, KODSDD, KODFLDD, KODSDF, and KODFLDF.

Although each of these subroutines have certain independent functions, it is probably only worthwhile to describe a select few so the reader can observe the basic operation of the program. The most useful illustrations of the program's operation are best given by a detailed examination of the subroutines DIFALGO, REORGA, NEWTSIM, and DECDJ.

Subroutine DIFALGO

The basic procedure of this differential algorithm is contained in the subroutine DIFALGO where the minimization technique is coordinated. Aside from whatever peripheral program that might be using the algorithm for some phase of its operations, the primary control in the program itself is in the subroutine DIFALGO. A detailed flow chart of this subroutine is illustrated in Figure 4.

After entering DIFALGO, the first step is the initialization of certain internal control variables, as well as an array variable necessary in later computations. Then, an interative loop is entered in which a prescribed number of steps toward the minimum will be taken, or until the minimum is reached within satisfactory tolerances. The value of the objective function and the constraint slack variables are next calculated by calling subroutines YOFX and FKOFX. Information from the latter is subsequently used to determine both the number of active and inactive constraints so the number of state variables can be determined. Control is then shifted to subroutine ARRAY for the first initial partition between state and decision variables, and active and inactive constraints. Calling the subroutines DYDX and DFDX provides the values of the objective function and constraint derivatives which are next used in the subroutine REORGA, which reorganizes this data according to the variable partition accomplished in subroutine ARRAY and then checks the determinant of the Jacobian matrix, $\underline{J}$, to insure non-singularity. Then, DIFALGO calls the subroutines KODYDD and KODYDF, which provide the values of the constrained derivatives of the objective function with respect

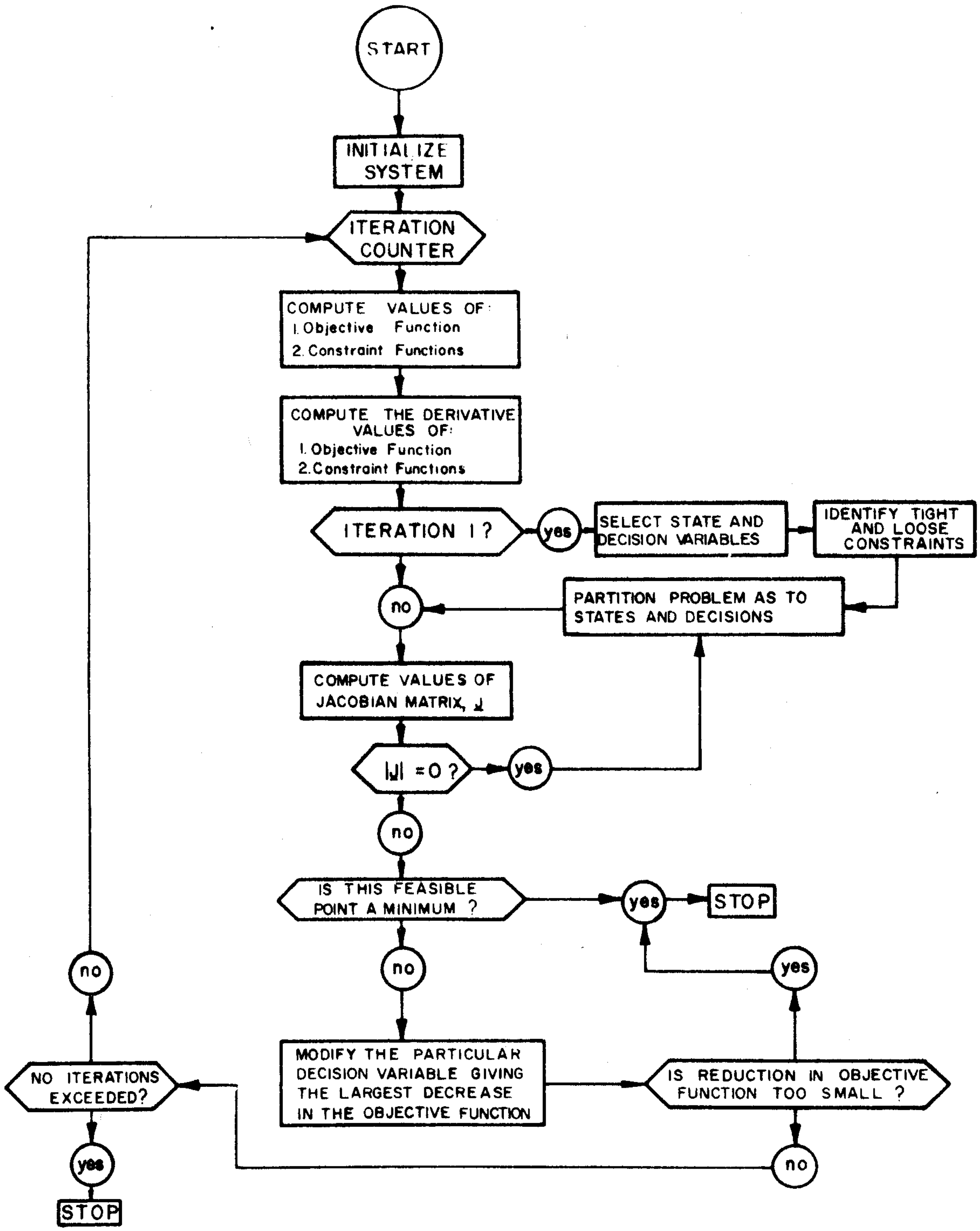

Figure 4. Illustrative flow chart of the subroutine DIFALGO.

to the decision and slack variables. These values are then used in subroutine JORK to determine which decision or slack variable is to be modified. The Kuhn-Tucker conditions are next checked; if they are satisfied, the procedure succeeded. Control is then passed to the appropriate change function (decrease d_p, DECDJ, increase d_p, INCDJ, or loosen a tight constraint, INCFT) where the step toward the optimum is taken and all problem boundaries are checked for violations. Finally, the program returns to the next iteration.

Subroutine REORGA

REORGA is essentially a bookkeeping and filing subroutine necessary to managing the continual changes that occur in the immediate structure of the problem. It is called not only from DIFALGO, but also from each step in the subroutines responsible for changing the decision and slack variables. A detailed flow chart of this subroutine is presented in Figure 5.

Upon the transfer of control to REORGA, the subroutine's first task is to relabel the derivatives of the objective function, active constraints, and inactive constraints with respect to the x variable defined in the problem formulation into derivatives of these parameters with respect to state, decision, and slack variables. Once this function has been completed, the subroutine JACOB1 is called where the Jacobian matrix is defined and its

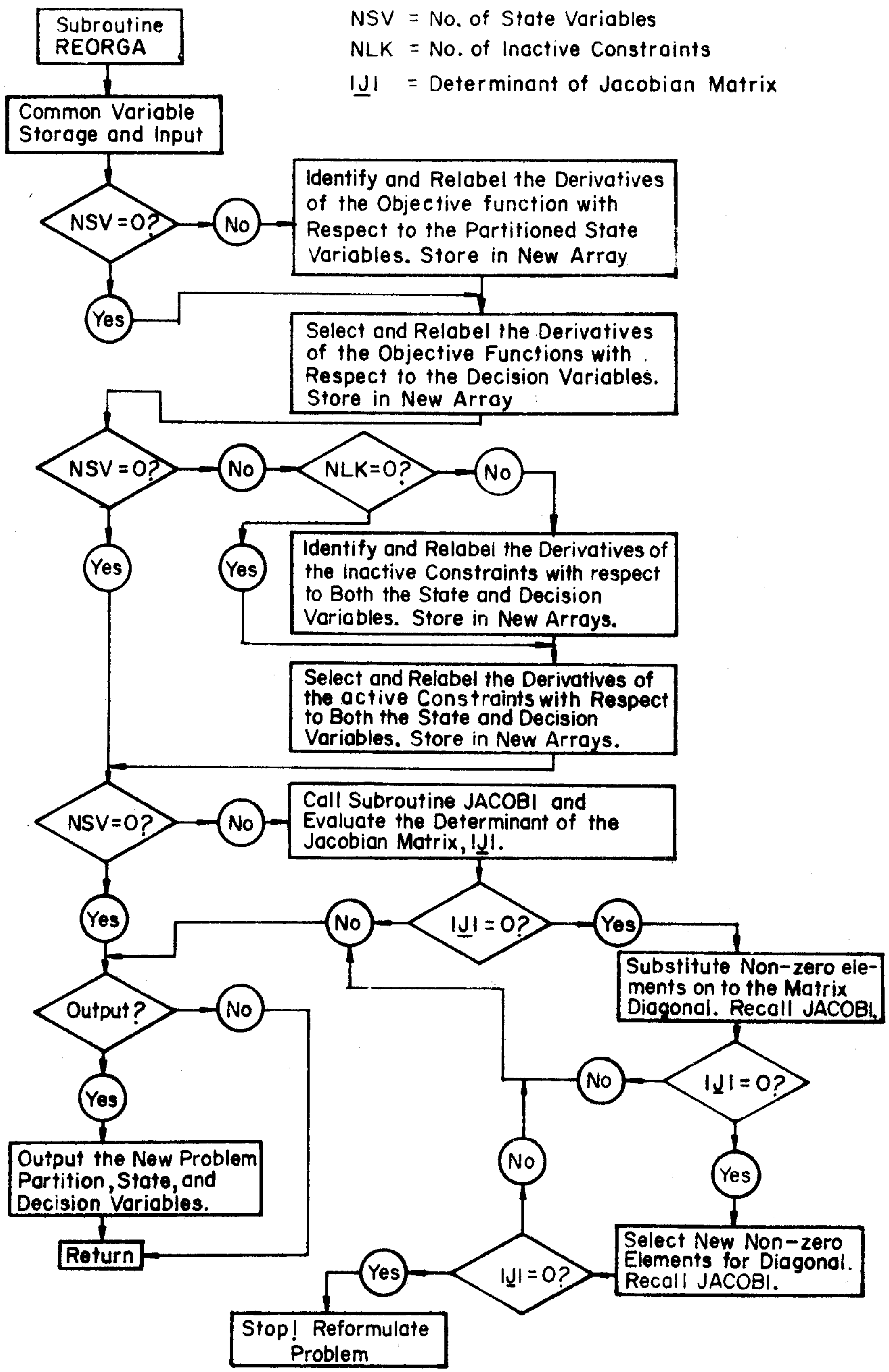

Figure 5. Illustrative flow chart of the subroutine REORGA.

determinant is evaluated. If this value is not zero, then REORGA concludes its function and control is returned. However, if for some reason the current partition between states and decisions yields a singular value for the Jacobian matrix, REORGA attempts to restructure the partition into a non-singular condition. In problems with many state variables, this may be an almost impossible requirement because of the enormous number of variable combinations possible. In REORGA, the best plan that could be thought of was one of trying to make all diagonal values in the matrix non-zero. Unfortunately, cases have been found where this is insufficient in which the problem definition needs to be re-evaluated. Generally, the diagonalization will provide a non-singular Jacobian matrix.

Subroutine NEWTSIM

Throughout this differential algorithm, systems of non-linear equations must be solved in order to determine the real values of the state variables. The procedure for accomplishing this is the so-called Newton-Raphson method, which is incorporated in subroutine NEWTSIM.

This procedure is derived by expanding Equation 2 in a Taylor Series and by ignoring the higher order terms:

$$\underline{f}(\underline{x})^{\nu} = \underline{f}(\underline{x})^{o} + (\nabla_{\underline{x}}\underline{f})\,\partial\underline{x} \quad . \quad . \quad . \quad . \quad . \qquad (32)$$

Then, noting that $\partial\underline{x}$ can be approximated by $x^{\nu} - x^{o}$, and rearranging terms:

$$\underline{x}^{\nu} = \underline{x}^{o} - (\nabla_{\underline{x}}\underline{f})^{-1}\ \underline{f}(\underline{x})^{o} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (33)$$

This recursive equation can then be used to solve the non-linear equations.

There are several problems with the Newton-Raphson method which demand attention in NEWTSIM. Occasionally, the system of equations being solved represent functions with several inflection points or nodules. In these situations, if the step size in a decision variable is too large, the procedure may converge on meaningless points. To combat this occurrence (which is often), the NEWTSIM subroutine is able to back up until a proper solution is obtained. An illustrative flow chart of this subroutine is shown in Fig. 6. In some cases, the procedure simply will not converge on a solution. Generally, this means a poor problem formulation, but if it occurs, the output subroutines are called and the program will stop.

Subroutine DECDJ

DECDJ is the subroutine in which the particular decision variable, d_p, is decreased. It, along with INCDJ and INCFT, is the basic component in this optimizing method, and it is by far the most complex. This subroutine has been flowcharted in Figure 7.

The first operation of DECDJ is to store the entering values of the decision variable, d_p, and the constrained derivative of the objective function, $\delta y/\delta d_p$. Then, subroutines calculating the constrained derivatives, $\delta s_i/\delta d_p$

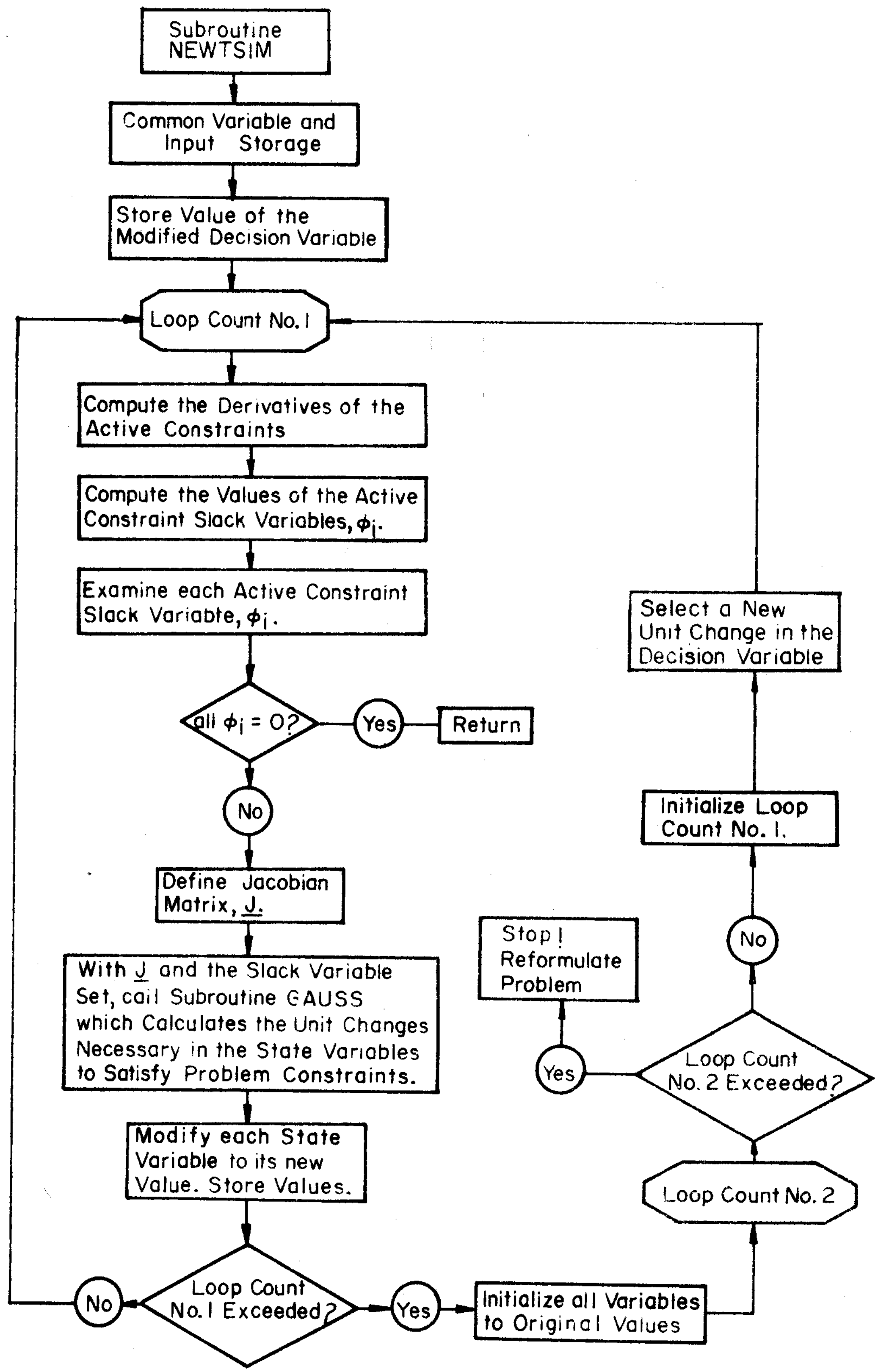

Figure 6. Flow chart of the subroutine NEWTSIM used to solve systems of non-linear equations.

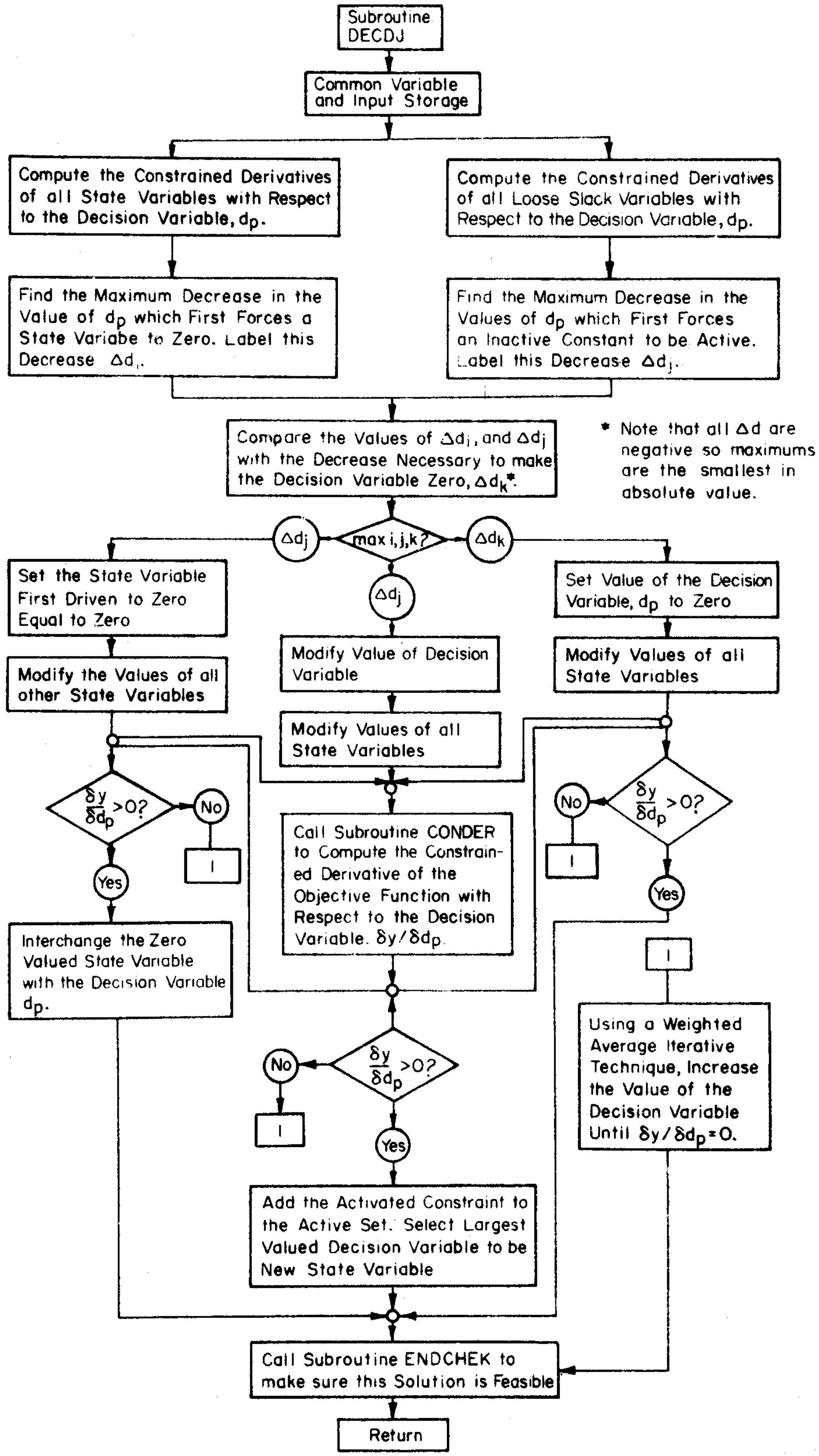

Figure 7. Illustrative flow chart of the subroutine DECDJ.

and $\delta f_{\ell}^{+}/\delta d_p$ are called. Next, the most limiting condition affecting the magnitude of the decrease in d_p is evaluated. Based upon this determination, the appropriate change in the decision variables is made and a new value for $\delta y/\delta d_p$ is calculated. If this value has become negative, the decrease has been too large and the procedure progressed past the minimum. When this occurs, the initial and new values of d_p and $\delta y/\delta d_p$ are used for a weighted average iterative procedure to adjust d_p to a value that results in $\delta y/\delta d_p$ being equal to zero. On the other hand, if these constrained derivatives of the objective function remain positive, the program continues. Three conditions occur:

1. d_p can be decreased to zero and no changes in the problem structure are necessary.
2. The decrease in the decision variable can force a state variable to zero. The particular state going to zero is already known, so the largest valued decision variable is interchanged with the state. Then, with the old state variable equal to zero, the new set of equations can be solved.
3. The decrease in d_p may result in a previously inactive constraint being tightened. In this situation, the number of state variables must be increased by one and a new variable and constraint partition determined.

At the conclusion of these adjustments, the subroutine ENDCHEK is called to make sure the new problem structure is a realistic one. If so, the control is passed first back to DECDJ and then to DIFALGO for a new iteration. If not, ENDCHEK redefines the structure and partition until they are satisfactory.

SECTION IV

URBAN WASTEWATER AND RECLAMATION MODEL

Introduction

The urban wastewater treatment and reclamation system is a complex network of unit operations, flow control points, and water quality objectives. Associated with each unit of treatment are the capital costs of construction and the costs of operating and maintaining these facilities. An analysis of these costs by Dérédec (1972) indicates that these facilities exhibit significant economies of scale, i.e., the marginal costs decrease with capacity. In a review of several sources of information, Dérédec (1972) summarized the costs of these facilities into useable cost functions and then compares the predicted values using these relationships to actual installations. These results indicated an accuracy of within about 10-20%. This accuracy is also sufficient for the purposes of this investigation.

In the model of the wastewater treatment system developed in this section, these relationships are used to reflect the costs of treating and reclaiming wastewater for recycling and achieving the standards set for urban effluents.

Formulation of Wastewater Treatment Model

The intent of the wastewater treatment model, illustrated in Figure 8, is to minimize the costs of the facilities subject to the water quality standards placed on the urban effluent and the water being recycled. The costs of recycled water are determined as the unit difference between the total system costs with and without recycling. Thus, by dividing the difference in these costs by the quantity of water to be reused, an average cost, or unit cost, for this water can be determined. The optimization of the wastewater treatment system minimizes the unit costs of recycled water, as well as the costs of achieving certain levels of pollutants in the released effluent.

For the purposes of this study, the water quality vector will be limited to two parameters: (1) the inorganic concentration of total dissolved solids, TDS; and (2) the commonly cited 5-day Biochemical Oxygen Demand, BOD. However, the cost functions represent treatment facilities which remove suspended solids, nitrates, phosphates, and other pollutants restricted by water pollution guidelines, as set by the regulatory agencies. The consideration of only two of these parameters by no means assumes that other quality criteria are unimportant. Instead, the intent of this limitation is to select two parameters that best characterize the overall quality of water. The evaluation of water management policies in the

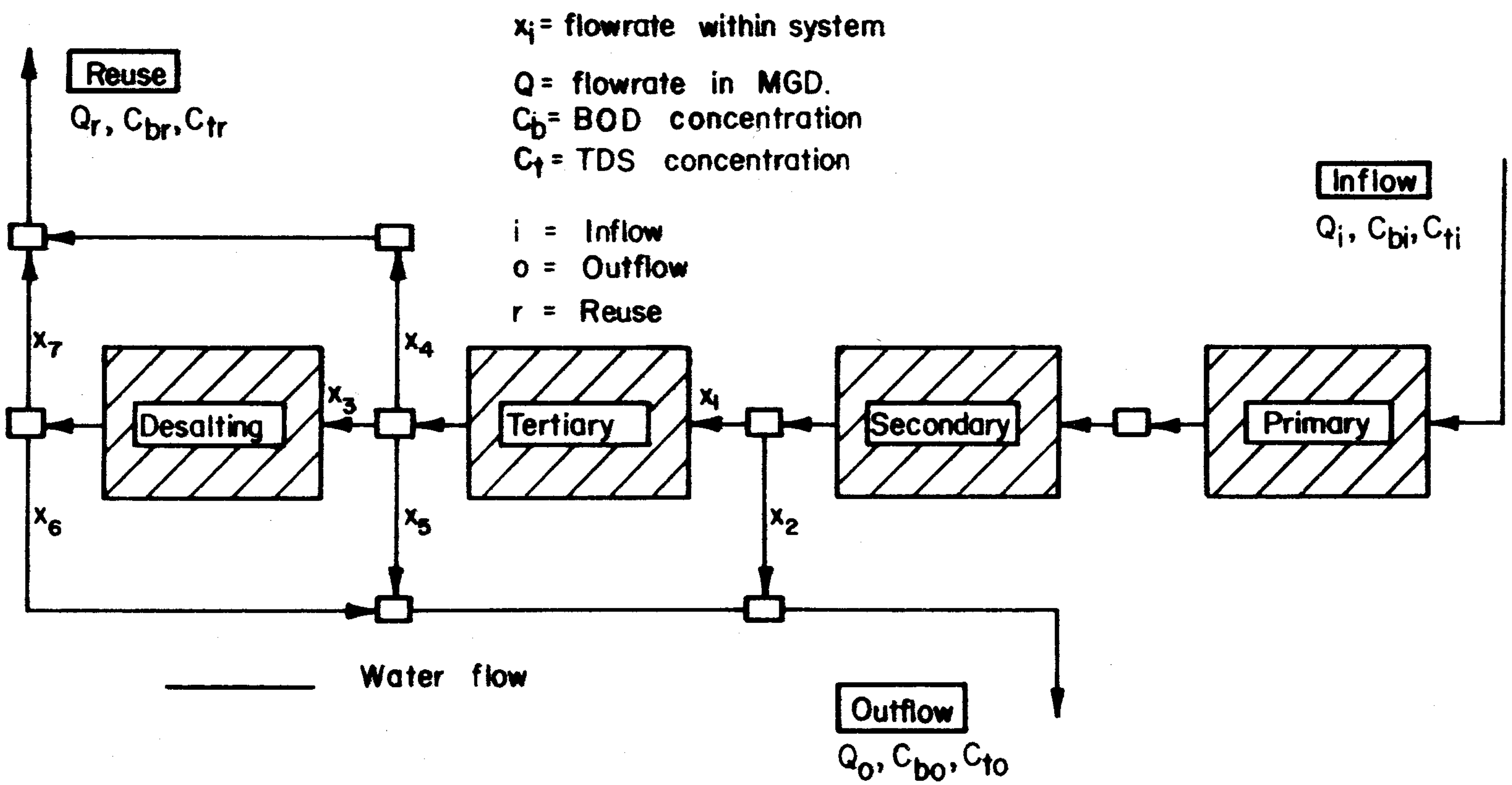

Figure 8. Schematic flow network of an urban wastewater treatment system.

urban environment requires that the interdependence between the sectors of the model be properly defined. As a result, TDS and BOD were selected as "indicators" of the effects that water quality in one part of the model have on the others. These variables are also widely used in design and monitoring and therefore are commonly measured.

Wastewater from the urban area is collected and sent first to the primary treatment. The quantity of these flows is defined as Q_i while their associated TDS and BOD concentrations are C_{ti} and C_{bi}, respectively. The primary effluent then becomes the influent to the secondary treatment phase. Upon concluding secondary treatment, the BOD levels are usually low enough to satisfy the 80% removal specified by present water quality standards (Nichols, Skogerboe, and Ward, 1972). However, as the water quality standards become more rigid, further treatment is necessary. Consequently, a decision must be made at this point as to how much water should be spilled into the effluent channels, X_2, and how much should be sent through tertiary treatment, X_1, in order to achieve a mix with a given level of BOD in the final urban effluent, C_{bo}. After tertiary treatment, three additional flow parameters must be decided upon: (1) the quantity of water released to the outflow, X_5, (2) the quantities released to the reuse system, X_4, and (3) the flows needing desalinization, X_3, to satisfy specified levels of TDS in both the outflow and the recycled water. The TDS constraints on the reuse system and outflows are defined as C_{tr} and C_{to}, respectively, which are met by mixing flows passing through the desalting process (X_6 and X_7), with the other flows. The flows in the wastewater model are regulated according to the water quality standards, physical system at each junction, and the quantities of outflow, Q_o, and reuse, Q_r.

The water quality objectives in this model function as constraints on the optimization procedure. Two constraints on the effluent water quality thus describe the restrictions on the two quality parameters, TDS and BOD. The functions can be written as,

$$X_2T_1 + (X_5 + X_6)C_{br} \leq Q_oC_{bo} \quad \dots\dots\dots \quad (34)$$

and,

$$(X_2 + X_5)C_{ti} + X_6T_2C_{ti} \leq Q_oC_{to} \quad \dots\dots\dots \quad (35)$$

in which T is the BOD concentration after secondary treatment in mg/l, T is the removal efficiency for the desalting process, in mg/l and C_{br} is the BOD concentration from tertiary treatment. The water quality constraints for the reuse segment can be written as,

$$X_4C_{ti} + X_7T_2C_{ti} \leq Q_rC_{tr} \quad \dots\dots\dots \quad (36)$$

representing only the concentrations of TDS since it is practical to assume that BOD levels after tertiary treatment would generally satisfy criteria for raw water supplies.

The interaction of flow rates and water quality, extends the mathematical non-linearity to the constraints of the preceding paragraph. Therefore, it is necessary to add physical flow constraints to the model to avoid unusual flows in the network. To begin, consider the outflow,

$$X_2 + X_5 + X_6 = Q_o \qquad (37)$$

and also the reuse phase:

$$X_4 + X_7 = Q_r \qquad (38)$$

In addition, the flow system must also be feasible at each decision junction in the model:

$$X_1 + X_2 = Q_i \qquad (39)$$

$$X_1 - X_3 - X_4 - X_5 = 0 \qquad (40)$$

$$X_3 - X_6 - X_7 = 0 \qquad (41)$$

The cost functions for each treatment process, along with these constraints, form the optimizing model of the wastewater treatment system.

Model Components

Primary Treatment

The first component of urban wastewater renovation, primary treatment, consists primarily of screening, grit removal, and primary clarification. Although these processes are quite often incorporated with secondary treatment, sufficient cost information exists in the literature to make the distinction.

Capital construction cost estimates for primary treatment facilities have been reported by several researchers. These estimating functions are helpful not only in establishing the costs of water quality control, but also in the planning of treatment plants themselves. Typically, these relationships have the form,

$$Y = aZ^{m} \quad \text{. . . . . (42)}$$

in which Y is the capital cost in millions of dollars and Z is the plant capacity in million gallons per day (mgd). Primary treatment as a whole is relatively subject to economics of scale as the exponential coefficient, m, usually ranges between 0.7 to 0.6. Smith (1968) states that in terms of 1967 dollars,

$$Y = 0.316Z^{0.71} \quad \text{. . . . . (43)}$$

while Shah and Reid (1970) propose a 1959 dollar value expression of:

$$Y = 0.331Z^{0.61} \quad \text{. . . . . (44)}$$

The operation and maintenance costs have also been of interest to managers, builders, and planners of wastewater treatment systems. The formulas which have been proposed by several investigators have the same general format as expressed in Equation 42. Michel (1970), for example, indicates that in 1967 dollars, the operation and maintenance costs are,

$$Y_o = 21{,}880Q^{0.59} \quad \text{. . . . . (45)}$$

where Y_o is the total annual operation and maintenance costs and Q is the average daily flow in mgd. However, these costs are more commonly expressed as costs per 1000 gallons treated, such as the work by Smith (1968) which uses 1967 dollars,

$$y = 4.47Z^{-0.17} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (46)$$

in which y is the operation and maintenance costs in cents per 1000 gallons.

Secondary Treatment

The principal components of the secondary treatment are most commonly either activiated sludge, rapid rate trickling filters, or slow rate trickling filters. For the purpose of this writing, the activated sludge process was selected primarily for its flexibility with respect to varying removal efficiencies. Although activated sludge appears to achieve greater removal efficiencies and more design flexibility than the other two, the costs are also somewhat higher. In a design context, the respective choice would be based on a more comprehensive analysis than is appropriate here.

The capital construction costs of building secondary treatment plants, while not indicating as large an economy with scale as encountered in the primary treatment plants, do nevertheless exhibit costs relationships with declining marginal costs with increased capacity. Shah and Reid (1970) state that in equivalents of 1959 dollars, the capital construction costs for these plants can be estimated from the following relationship,

$$Y = 2.48 \times 10^{-4} \; (PE)^{0.47} \; Z^{0.22} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (47)$$

where PE is the Population Equivalent of the organic loading expressed as,

$$PE = \frac{8.34\ QC_{bi}}{b} \quad . . . . \quad (48)$$

in which Q is the average daily flow in mgd, C_{bi} is the Biochemical Oxygen Demand (BOD) concentration of the flows in mg/l, and b is a constant, usually 0.17 lb of BOD per capita per day. Smith (1968) also presents an estimate of activated sludge plant costs:

$$Y = 0.58\ Z^{0.80} \quad . . . . \quad (49)$$

Of some additional interest is Shah and Reids' (1970) estimate of the ccsts of the activated sludge unit itself. This relationship, having the same format as Equation 47, is given in 1959 dollars as:

$$Y = 5.1 \times 10^{-3}\ PE^{0.46}\ Z^{0.36} \quad . . . . \quad (50)$$

The costs associated with operating and maintaining secondary treatment plants are listed in several sources. For example, Michel (1970) suggests three relationships for these costs:

$$Y_o = 3.16 \times 10^{4}\ Q^{0.73} \quad . . . . \quad (51)$$

$$Y_o = 28.2\ PE^{0.75} \quad . . . . \quad (52)$$

$$y = 9.02\ Z^{-0.107} \quad . . . . \quad (53)$$

<u>Tertiary Treatment</u>

Advances in wastewater treatment have led to several demonstrations of the feasibility of adding tertiary treatment to existing primary, secondary treatment facilities for further removal of waterborne contaminants (Evans and Wilson, 1972). Such advances have been prompted by several mounting crises. First, the need for water by municipalities, industries, and agriculture is outstripping the supplies of natural waters under existing inefficient practices. Secondly, 90% removal of BOD, for example, is not considered satisfactory to continually insure public safety and palatability (Fair, Geyer, and Okun, 1968). And finally, the advent of new pollutants in conjunction with mans' increasing life span subjects him to extended exposures to chemicals with yet uncertain results (Civil Engineering, 1972). Since very few if any existing primary-secondary wastewater treatment facilities can meet reduced pollution levels, or support a "zero level pollution" philosophy for urban effluents, tertiary treatment will become as necessary in the near future as secondary treatment is now.

In this writing, tertiary treatment will consist of flocculation, lime treatment, and sedimentation; granular carbon adsorption; and ammonia stripping. Although tertiary treatment has found only limited application to date, sufficient testing has been completed to generate general cost functions.

The capital costs of tertiary treatment can be found in several sources. Smith (1968) suggests that in terms of 1967 dollars, the following relationships can be employed:

$$Y = 0.05\ Z^{0.89} \qquad (54)$$

for flocculation, lime treatment, and sedimentation,

$$Y = .398\ Z^{0.65} \qquad (55)$$

for grannular carbon adsorption and,

$$Y = .0398\ Z^{0.90} \quad \ldots \quad (56)$$

for ammonia stripping. Barnard and Eckenfelder (1970) also give an estimating formula for granular carbon adsorption in 1959 dollars which will be included here for comparison:

$$Y = 0.20\ Z^{0.86} \quad \ldots \quad (57)$$

The costs of operating and maintaining tertiary plants are also listed by Smith (1968) in terms of 1967 dollars:

$$y = 2.99\ Z^{-0.038} \quad \ldots \quad (58)$$

for flocculation, lime treatment and sedimentation,

$$y = 10\ Z^{-0.28} \quad \ldots \quad (59)$$

for granular carbon adsorption and,

$$\begin{aligned} y &= 11.58\ Z^{-0.3} \quad Z \leq 3 \text{ mgd} \\ y &= 1.2\ Z^{-0.04} \quad Z > 3 \text{ mgd} \end{aligned} \quad \ldots \quad (60)$$

for ammonia stripping.

Desalting

The removal of salts from seawater and brackish waters has been under close examination for some time as a source for supplemental water supplies (White, 1971). The limiting factor to date has been the high costs as compared to other water sources. Among the promising techniques that have been developed, either electrodialysis, reverse osmosis, or a combination of these two methods seems to be the best suited for reclamation of urban wastewater (Dykstra, 1968). Again the flexibility with regards to removal efficiencies prompted the selection of electrodialysis for this model.

The capital construction costs for desalting plants has been suggested by Smith (1968) to be,

$$Y = 0.51\ Z^{0.67} \qquad (61)$$

and by Rambow (Cited by Dérédec, 1972),

$$Y = 0.219\ Z^{0.66} \qquad (62)$$

which are also in terms of 1967 dollars. The first equation is for a 90% TDS removal and the second is for a removal of 500 mg/l.

The same two sources supplying capital cost information also suggest the following operation and maintenance costs:

Smith (1968) $\quad y = 47.94\ Z^{-.21}$ . . . . . . (63)

Rambow $\quad y = 10.2\ Z^{-0.12}$ . . . . . . (64)

indicating significant variation.

A single-stage electrodialysis process applied in this model is assumed to have a removal efficiency of about 40%. Therefore, the costs suggested by Smith (1968) would be for a four-stage demineralization system.

Operation of Wastewater Treatment Model

In order to provide the reader with a clearer understanding of the urban wastewater treatment model and illustrate its use in evaluating optimal policies in the overall urban water system, it is useful to examine some of the types of results generated by the wastewater treatment model.

The cost functions presented in Equations 34 to 64 are in the present-worth dollar value set by the original authors. These relationships were multiplied by an adjustment factor to convert all of them to 1970 dollar values and then a set of results were generated to delineate the basic characteristics of the system.

The first characteristic of interest is the effects varying effluent quality standards have on the unit costs of recycled water. An illustration of this influence, shown in Figure 9, represents a system with a reuse capacity (Q_r) of 30 mgd, and a fixed effluent TDS standard (C_{to}) of 600 mg/l. The unit costs (present-worth) are shown as a function of BOD standards on the outflows (C_{bo}) with four levels of TDS concentrations in the recycled flows. It is interesting to observe the curves at an abcissa value of about 10 mg/l. At this point, the unit costs for the higher limits of TDS in the reuse become negative. This characteristic illustrates that when water quality standards on the outflow are sufficiently restrictive, the capacity of the desalting plant (and therefore the costs) is larger than if the system would permit some flows to be diverted to reuse at a poorer quality. The effect therefore of increasingly stringent standards on urban effluents is to substantially enhance the feasibility of reclaiming and reusing wastewaters.

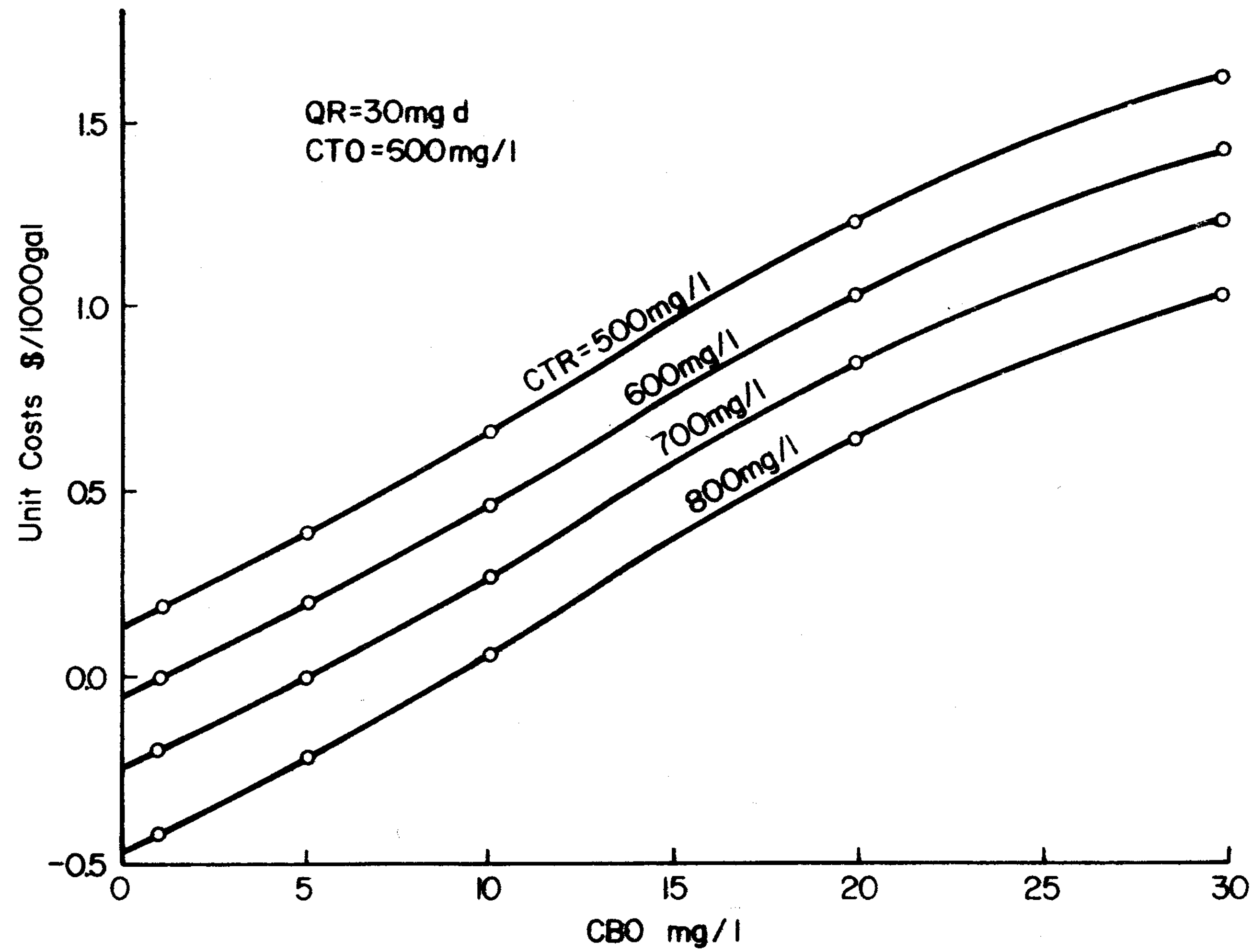

Figure 9. Unit costs of recycled wastewater under varying water quality standards.

Another interesting attribute of the wastewater treatment model is shown in the plots of Figure 10. The economics of scale in the reuse system are shown to be affected by the water quality constraints on the outflow. Results from two analyses in which the outflow BOD concentration is restricted to a value of 10 mg/l are plotted. In the upper segment, the TDS standard is fixed at a value of 800 mg/l and the unit costs for a series of reuse capacities are computed, as functions of the concentrations of TDS in the recycled water. It is observable that larger capacities are much less affected by the level of TDS in the reuse than are the smaller values. Furthermore, the economy of scale is clearly evident with the larger systems having unit costs that are substantially less at the lower concentrations of TDS in the recycled water. The curves in the lower segment have the same basic characteristics as the upper curves, except that the outflow TDS standard is set at 500 mg/l. It is interesting to note that the scale effects are almost eleminated.

Each of the Figures 9 and 10 demonstrate the impact that increasingly rigid water quality standards have on the economic feasibility of reusing some urban effluents as supplemental water supplies. The wastewater treatment model discussed thus far in this chapter is employed in the overall model to optimize the water supply policies in the urban water supply and distribution segments. However, aside from the water quality constraints imposed on the

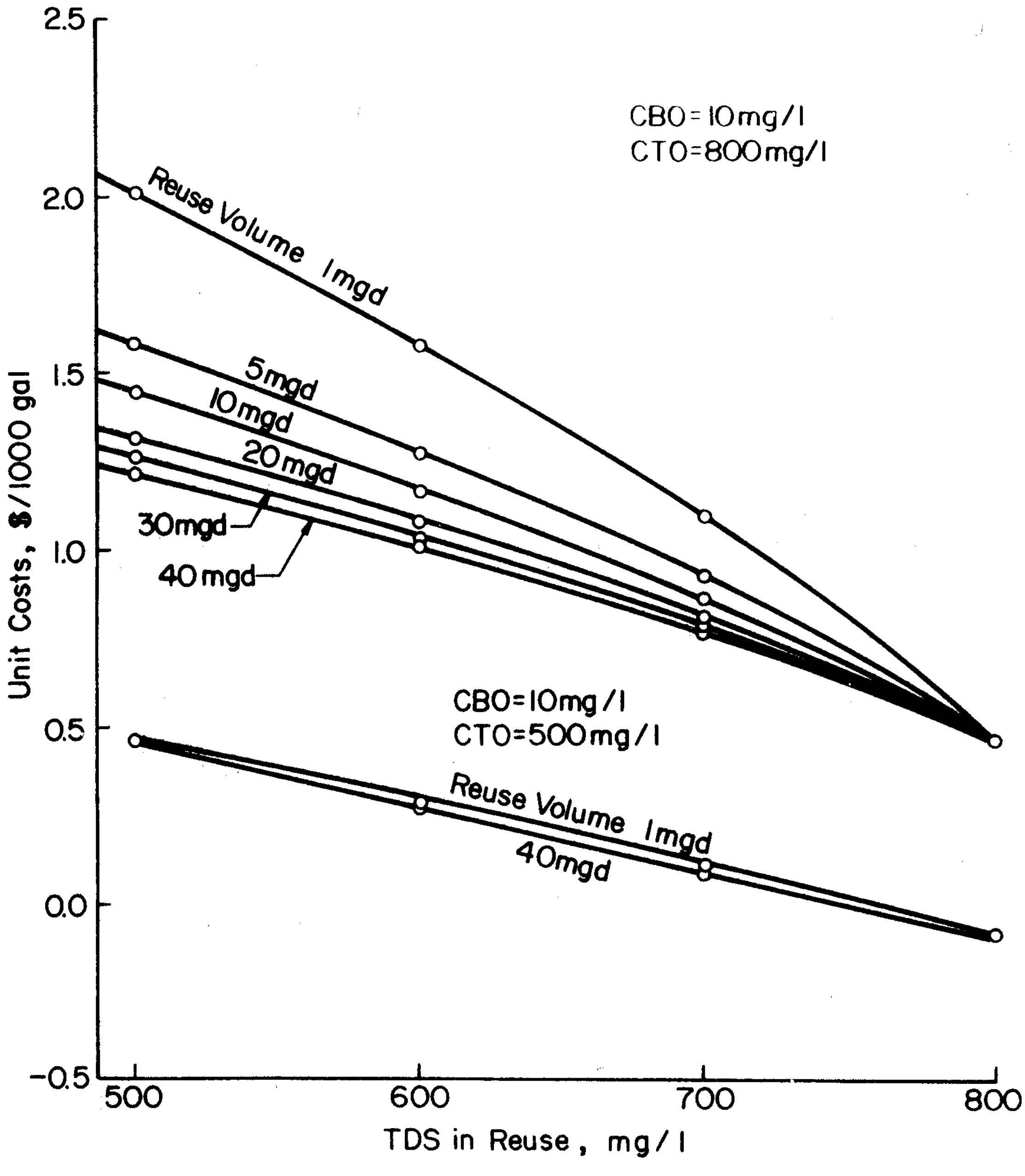

Figure 10. Effects of varying water quality standards on the unit costs and economies of scale of recycled wastewater.

urban effluent, the optimal values of reuse, its TDS concentrations, and several other important variables are not known during the initial stages of the problem solution. Consequently, the context of decomposition is used to iteratively improve the solution until the optimal policy is formulated. To do this efficiently, only one curve in Figure 10 is generated and a polynomial regression of the function is calculated. This is done initially for assumed variable values in the wastewater treatment system. Then the water supply and distribution model is optimized. New values of input to the wastewater treatment model are now known and another iteration is made. This process is repeated until no further refinement is possible.

Until this point, all the cost functions have been in terms of total present-worth. However, in the examination of the optimal water management policies, annual costs are more commonly employed. To facilitate these requirements, the present-worth calcualtions in the preceding paragraphs have been transformed into a uniform series of annual costs. To do this, it has also been necessary to add the interest costs to the function. This procedure has been accomplished in Figures 11 and 12 in order to demonstrate the final results gained from this submodel.

Before proceeding with a description of the water supply and distribution model, some comment regarding the assumptions made to formulate the wastewater treatment model should be made. First, no attempt has been made to model

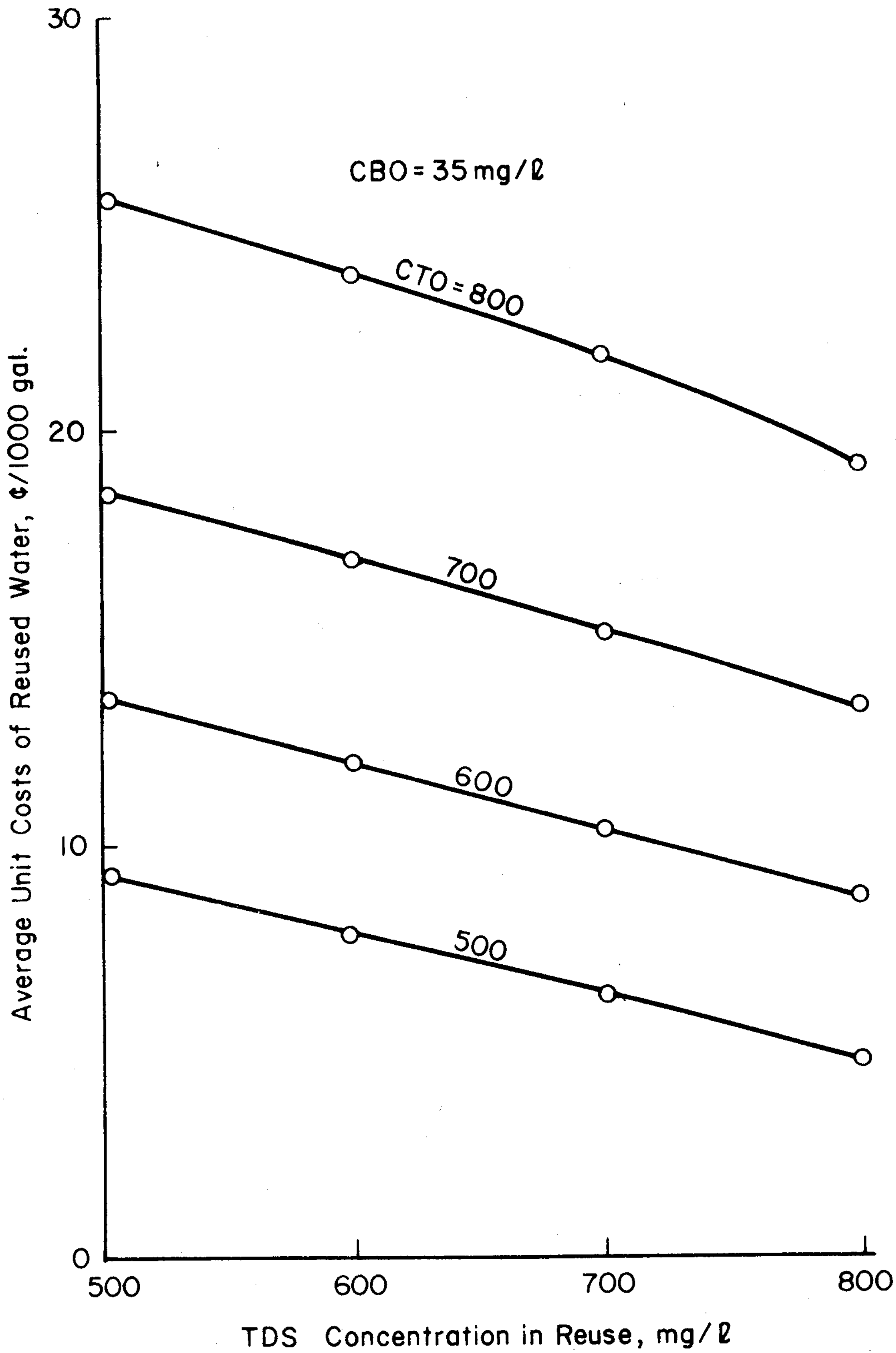

Figure 11. Average unit costs for reused wastewater for a BOD limit on the urban effluent, CBO, of 35 mg/l and various levels of effluent TDS, CTO.

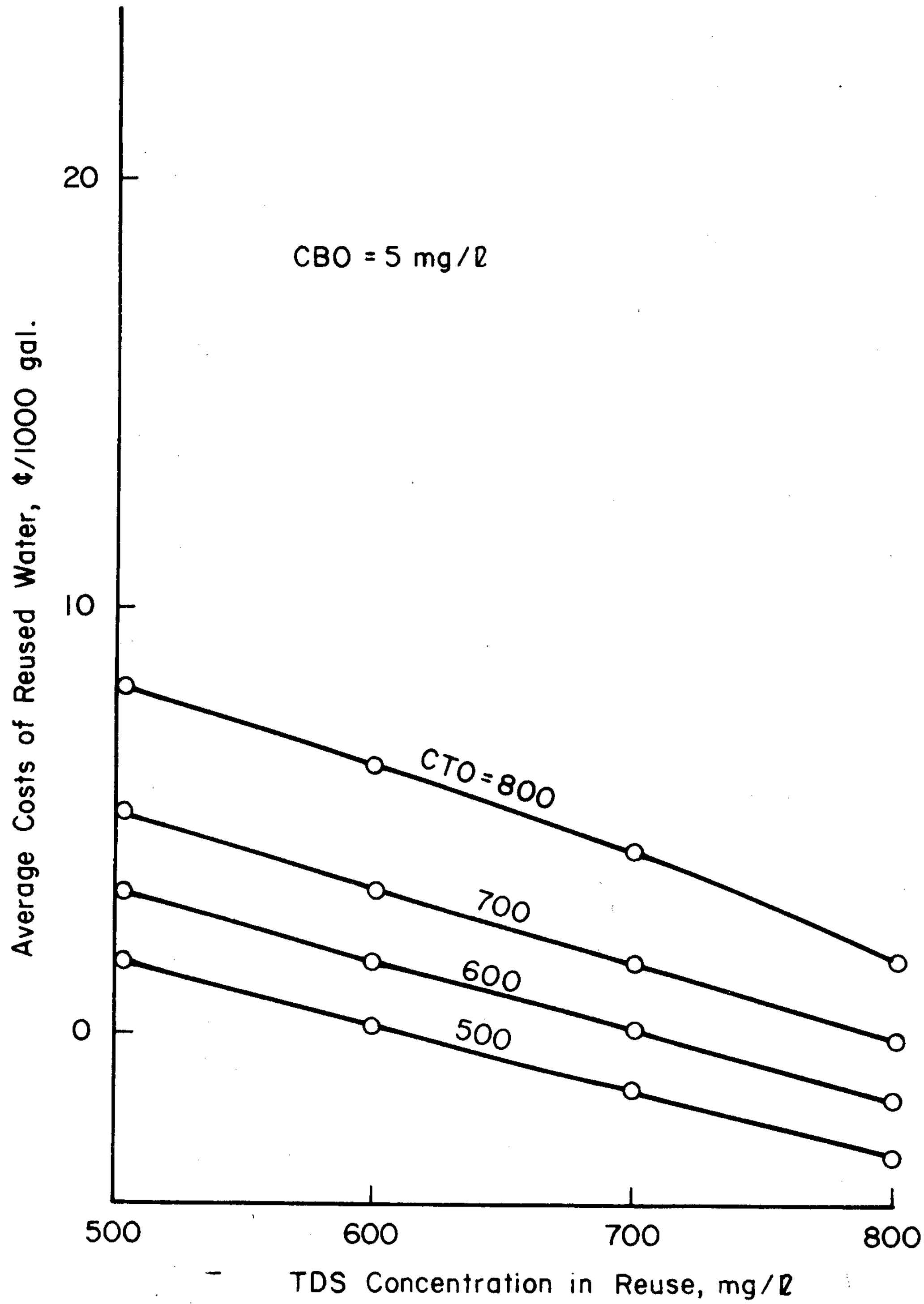

Figure 12. Average unit costs for reused wastewater for a BOD limit on the urban effluent, CBO, of 5 mg/1 and various levels of effluent TDS, CTO.

the numerous alternative treatment schemes which may improve the cost-effectiveness of this system. For example, the use of polymers in primary treatment have been shown to significantly improve the primary removal efficiency in certain circumstances (Henningson, et. al, 1970). Justification for this assumption is because cost information is not accessible at this time in the general literature from which cost functions could be formulated for inclusion in this section.

SECTION V

THE URBAN WATER SYSTEM MODEL

Introduction

In the preceding section, the urban wastewater treatment and reclamation system model was developed prepatory to its use here. The water supply and distribution system model, defined in this section, focuses on the analysis of water supply alternatives. However, because recycling is an integral part of urban water supply, the linkage between the two systems is brought to light.

The scope and format of the urban water system model derived and explained in this section follows the "limited purpose model" concept. The intent of this development is to provide the mathematical description of the broad and macroscopic characteristics of urban water systems and evaluate the effects of changing institutional constraints, such as water quality goals, on the optimal water management policies. Consequently, the model is less useful as a design or capacity determining tool as it is for delineating and comparing various planning and management alternatives. By limiting the scope of the model in this manner, and avoiding the entangling detail of the exact nature of the flow networks, the model can be general in nature and adopted to other areas with a minimum of modification.

The basic nature of the model as it is operated begins by combining the water supply alternatives with the distribution system, leading to the individual urban demands, as illustrated in Figure 13. This leaves the urban water system model in two distinct components which are then conjunctively solved to optimize the complete network. The procedure involves the following three steps:

(1) A quantity and quality of water needed for reuse from the reclaimed wastewater treatment system is assumed and unit costs for this water are computed for a range of TDS concentrations. Then a polynomial regression of these data points is computed giving the unit costs of recycled water as a function of its TDS concentrations.

(2) Using the unit costs previously determined for recycled water, the model optimizes the water supply and distribution subsystem to evaluate the optimal water management policy.

(3) The assumed values of reuse are contrasted with the quantities actually employed in the optimal plan. If these values differ markedly, new values of reuse parameters are assumed and the process repeated.

In addition to recycling wastewater, alternative water supplies such as interbasin water transfers,

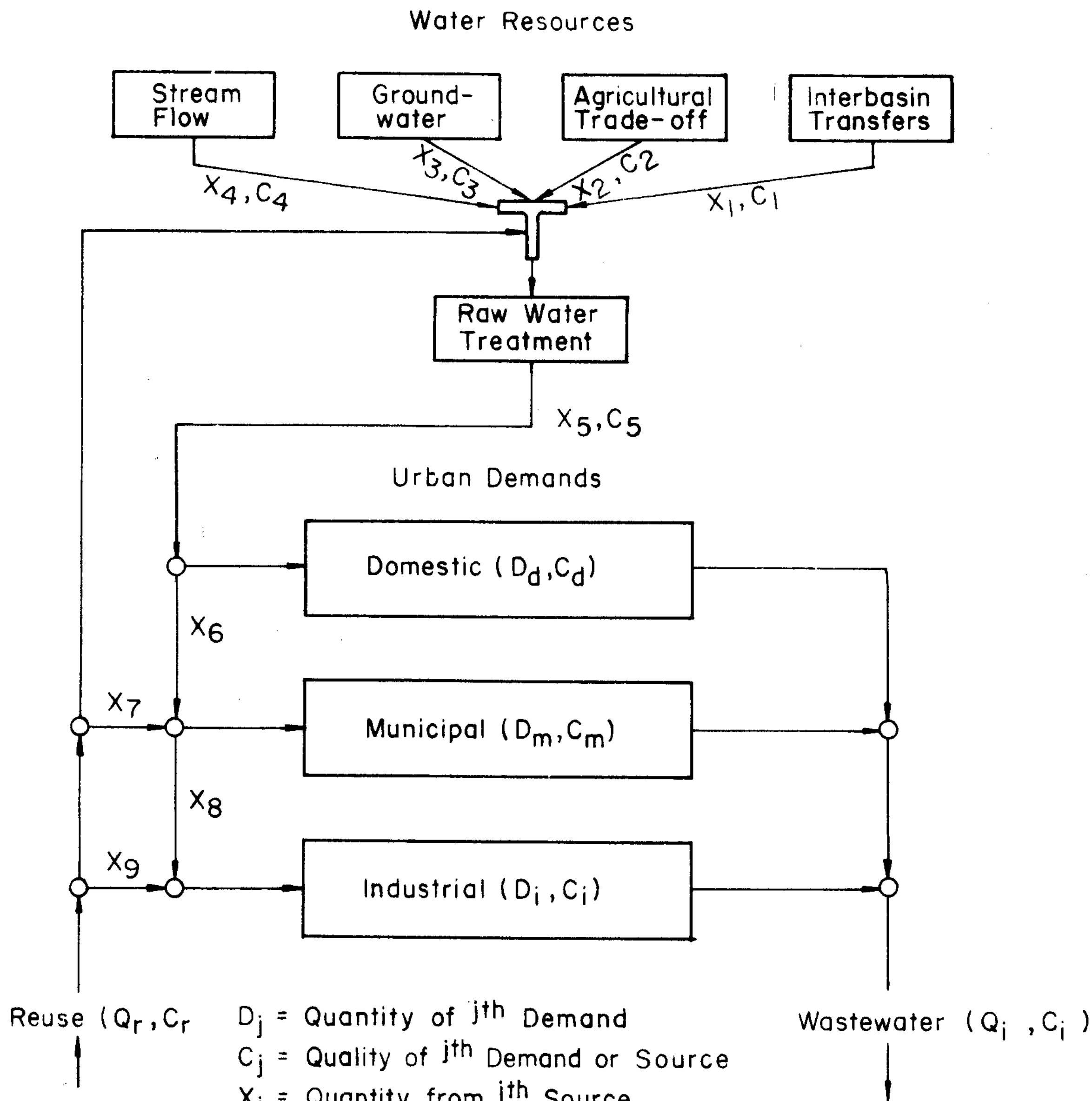

Figure 13. Schematic diagram of the urban water supply and distribution system model.

groundwater, agricultural right aquisition and transfer, and, of course, in-basin stream flows are available. The characteristics of these resources are complex, but have been extensively investigated. Although a thorough review of such characteristics is infeasible, some general comments are helpful in realizing the potential uses and extensions of the urban water system model.

Water Sources

Stream Flows

Water resources generated within the hydrologic unit encompassing the urban demand consist primarily of stream flows. Reservoirs, diversion works, and conveyance systems transform the stochastic variations in these flows into demand frequency water supplies. Such supplies were developed in competition with other interests, such as the agricultural and mining enterprises. Because urban areas have generally been junior appropriators, they have been forced to develop additional alternative water sources to meet growing needs.

The competitive characteristic of over-appropriated water supplies has prompted numerous attempts to impart an economic value or price to the flows. Such studies seem to indicate that water values are higher than existing prices because of the protective influence of the water right system. In order to avoid the surrounding controversy regarding the planning values these resources should assume, existing rights are valued at estimated current costs. Then, in the model operation to be described later, an attempt is made to indirectly value additional stream flows according to actual worth. A more specific description of this analysis will be deferred until later in this writing.

Interbasin Transfers

Transferring water resources from one river basin to another is among the most feasible alternatives for supplying the needs of water-short areas. However, most proposals for these transfers in the West have resulted in political conflict because of the high value of the scarce resource. Such conflicts have been observed in both the Congress and State legislatures, in the Federal agencies, in the association of Federal and State governments, in the courts, and even on the canal or reservoir banks (National Water Commission, 1972). The conflicts have also extended to various interest groups, such as conservationists and the large urban centers.

The costs of interbasin transfers are three dimensional in nature. First, the capital outlays for construction of the necessary facilities to import the water are very large. However, unlike the water treatment plants, these facilities are relatively permanent in nature. For example, the structures involved may include reservoirs, tunnels, canals, and diversion and control structures. Since the expected life of these facilities extend beyond the planning horizon, they may be easily financed for as much as 50 years or more, which substantially reduces the annual costs. The second economic consideration regarding interbasin transfers is the operation and maintenance costs. Again, the permanent nature of the system is characterized by a relative freedom from maintenance and thus have low operation and maintenance costs. And finally, the third dimension of costs associated with water importation is the externalities, or costs to downstream water users as a result of the water transfer. Externalities are difficult to quantify and require a more specific analysis than is permissible here.

External costs cannot be ignored in regional water resource management, but are difficult to incorporate into local planning efforts. It would appear plausible therefore, to suggest further investigations regarding the external effects of the importations and coordinate water developments on a state level.

Because interbasin transfers are being considered on such a large scale in the western states, it would be helpful for the cost versus capacity relationship to be incorporated in a model such as this. However, such an analysis has not heretofore been completed in sufficient detail to include in this description. Thus, the model as presently formulated uses only unit costs based on data available in the various project reports.

Agricultural Water Transfers

Once a pattern of use has been initiated in a river basin, changes in water usage at one point in the system are reflected elsewhere in the system. Such changes may diminish both the quantity and quality of the water resource available to other users. In many cases, these factors may not be critical during normal years because even though the annual flow is completely appropriated, it may not be fully utilized. However, during low-flow periods, or as the demands increase, the damages resulting from changes in use practices may become significant.

As these general trends continue, urban growth may expand the demands for water beyond the safe annual yield of the urban areas' water rights. Historically, two alternatives were immediately obvious: (1) interbasin transfers; and (2) agricultural water right acquisition. Initially, the second alternative was pursued. Cities began buying agricultural water rights and then filing for a change in the points of diversion, thus initiating the transfer of waters within the basin. The attempts to do so along the eastern slope of Colorado were almost futile. In fact, Hartman and Seastone (1970) found in an examination of records in the State Engineer's office, that only 33 cases involving transfer of 22 second-feet from agricultural to municipal use have been successfully completed in the state of Colorado (excluding Denver). Of these, only 9 have been completed since 1930. The poor success in transferring water from agricultural to municipal use according to Hartman and Seastone (1970) is the uncertainty of court interpretations. As a result, the major cities in eastern Colorado have turned primarily to interbasin transfers.

Although a brief discussion of the process of transferring agricultural water to municipal use has been presented, it serves only to justify the approach taken in the urban water system model presented herein. It is concluded that attempting to directly incorporate this alternative in the model would be infeasible because of its largely institutional nature. As a result, water obtained from agricultural transfers will be omitted from the model, but an analysis will be presented which indicates the value of such transfers to the urban users. In this manner, an estimate of the costs of this alternative as a potential water supply can be generated.

Groundwater

The analysis of groundwater is an important part of total water resource management. Yet, this water resource has undoubtedly one of the weakest institutional structures for optimal management of any phase of the developed hydrologic cycle. Nevertheless, the investigation of water as it exists in subsurface strata has been extensive. In fact, the topic of interest in this study, alternative water supply policies for arid urban areas, is directly related to the exhaustive analysis which has been performed to optimize the conjuctive use of groundwater and surface-water resources.

The use of groundwater for urban water supplies has several distinct advantages over other possibilities. First, the waters located in confined or unconfined aquifers are nearly unaffected by seasonal variations in temperature. Secondly, the relatively slow water movements in these aquifers results in seasonal variations in runoff being almost completely dampened. Thus, the supply remains essentially uniform over a season. Third, groundwater flows are not subject to evaporation generally and outflows are small, so that the groundwater basin acts as a minimal loss reservoir. And finally, water quality characteristics of groundwater tend to be constant with time. Although groundwater pollution is a possibility, usually fresh water supplies when located remain a good source of water supplies.

In this application of the urban water system model, groundwater is not considered alternative because of the unconfined nature of the stream-aquifer system in the South Platte drainage. If application of the model in future investigations needs to include groundwater costs, the necessary changes can be made in the objective function and constraints with little difficulty.

Water Distribution Network

Within the framework of the urban water use, three broad categories of use exist: (1) domestic uses; (2) municipal uses; and (3) industrial uses (Flack, 1971). Although some areas have incorporated separate distribution systems to serve each of these needs, the general case may only find distinction in rates, seasonal variations, and location. Nevertheless, each of these classifications have individual water quality limits, growth rates affected by independent parameters, and a differing importance relative to the preferences of the local populace.

Albertson, Taylor, and Tucker (1971) also make this separation in the urban water utilization subsystem in order to make their model more amenable to a systems analysis approach in evaluating alternative decisions. However, their primary thrust and that of other writers in the publication, is the need for the systematic approach. This attitude has been adopted in this writing as well. In this section, the purpose is to note a few of the characteristics of each of the three classifications of water use and discuss the assumptions employed in this model as it investigates the various potential decision strategies.

Domestic Water Uses

The first objective of a municipal water supply system is to serve the immediate needs of the population. When water supplies are insufficient to meet all potential demands, one after another of the less important uses may be restricted. In the ultimate limitation, water would only be available for such things as drinking, bathing, and cooking. Thus, the domestic water uses are the first priorities of the urban water system.

It has been traditional to lump outside-the-house uses such as lawn or tree watering with the other domestic uses. However, these uses are also subject to rationing during critical periods. Consequently, lawn and tree watering have been deleted from the domestic category and placed with those considered as municipal uses. Such an assumption may be infeasible due to the physical structure of the distribution system. Nevertheless, it does represent the extreme limit of separation between the two uses, and it is, therefore, of interest in this analysis.

Domestic water uses not only maintain first priority on the water supply, but its water quality as well. Many other uses may be restricted by the concentration of total dissolved solids, suspended solids, phosphates, nitrates, heavey metals, or organisms, to name only a few. Domestic water uses have limits on nearly every water quality parameter currently used. As a result, treatment for such uses must be more extensive. In the model proposed herein, the philosophy has been adopted to maintain water quality limits for the domestic use at their present level, even though these levels may not be as high as the tolerable limits. The basis on which such an assumption is based is the fact that drinking water standards are upper limits and not desirable if reduced water pollutants can be maintained.

Municipal Water Uses

Although the fire protection water needs may be nearly as important in an urban area as the domestic needs from purely a survival viewpoint, other municipal water uses such as park, golf course, lawn, and tree irrigations may be important to the living environment. Such uses are generally regarded as supplemental to the enjoyment of living in the urban area.

A great deal has been said concerning the difficult decisions to be made during periods of water shortage, but the cause of the shortage has not been stated. Certainly a man stranded in a life raft in the middle of the ocean is as water short as another lost in the Sahara Desert. It can therefore be concluded that an important method of extending water supplies by more efficient use would be to rearrange the urban water system in such a manner as to distribute water on the basis of quality needs rather than quantity aspects alone. By dividing the urban water use and extracting municipal uses to be served by a poorer quality water, especially with regards to reuse, more water is available to the needs with more restrictive quality characteristics.

Industrial Water Uses

Industrial water use, or commercial use, is defined somewhat differently in this study than its real meaning. For the purposes of this study, industrial uses are those being filled by the urban water system. Fair, Geyer, and Okun (1968) states that more than 60% of the industrial needs in the United States are met by internal reuse. Thus, only 40% of the industrial requirements are on the average served by the metropolitan water system.

If domestic needs exist because people need water to survive, and municipal needs exist because people demand an enjoyable living environment, then industrial needs exists because people are in the urban area to work in the direct and indirect needs of industry. In other words, people live in cities because of industrial concentration and the needs to support industrial jobs with services. Therefore, the priority on industrial water is second to the domestic uses.

The water quality constraints on industrial water use are as varied as the nature of the industries themselves. In this investigation, industrial quality requirements have been limited to the maximum suggested for public potable supplies.

Model Formulation

The model proposed in this thesis has been formulated in the context of long range planning, but from a unique viewpoint. By simplifying and generalizing the model beyond the intricate details of either physical or institutional structures of any specific location, two central advantages are gained:

(1) The optimal policies derived and the alternatives evaluated indicate decision making on the scale of planning alternatives of water supplies.

(2) Only those institutional constraints violated by cptimal strategies are identified. Thus, those which effect future decisions are valued in the sense that they can be priced by comparing with and without analyses.

Model Constraints

The urban water supply and distribution system model is bounded and operated by two major types of constraints: (1) water quality constraints; and (2) water flow constraints. The water quality constraints in effect place an upper limit on the quality flows diverted to each of the three use categories. It has been assumed that tertiary treatment would be necessary for any quantities of water recycled since public contact, occurring in all use categories, would forbid objectionable odors and potential health hazards resulting from the higher levels of BOD. In addition, it is unlikely that desalting could be feasibly accomplished if the suspended solids were limited only by primary and secondary wastewater treatment.

To begin, the quantity of water treated in the municipal raw water treatment system and its associated quality are defined,

$$X_5 = \sum_{j=1}^{4} X_j + (Q_r - X_7 - X_9) \quad \ldots \ldots \ldots \quad (65)$$

$$C_5 = \frac{\sum_{j=1}^{4} X_j C_j + (Q_r - X_7 - X_9)(C_{tr})}{D_d + X_6} \quad \ldots \ldots \quad (66)$$

in which:

X_j = quantity of water from source j

X_7, & X_9 = quantities of water recycled directly to municipal and industrial uses

Q_r = total quantity of water from reclaimed wastewater

C_j = associated water quality (TDS) from respective sources

C_{tr} = the TDS concentrations in the reuse water

D_d = domestic demand

Then for each water use, an upper limit on TDS can be defined,

$$C_d - C_5 \geq 0 \quad \ldots \ldots \ldots \ldots \ldots \quad (67)$$

$$C_m - \left[\frac{X_6 C_5 + X_7 C_{tr}}{D_m + X_8}\right] \geq 0 \quad \ldots \ldots \ldots \ldots \quad (68)$$

$$C_i - \left[\frac{X_8 C_5 + X_9 C_{tr}}{D_i}\right] \geq 0 \quad \ldots \ldots \ldots \ldots \quad (69)$$

$$C_{ti} = \phi(C_d, C_m, C_i) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (70)$$

where

C_d, C_m, C_i = maximum allowable TDS for domestic, municipal, and industrial uses.

C_{ti} = water quality of wastewater as a function of the water quality supplied to the demands.

D_m and D_i = municipal and industrial demands.

These constraints can be varied over an applicable range to determine the costs associated with delivering water to users of various quality.

The physical flow constraints become necessary not only to insure each demand is satisfied, but also to maintain feasible solutions. Because the model operates in an optimization format, it is necessary to provide for realistic solutions as the model minimizes costs. These constraints on the system can be written:

$$D_d + D_m + D_i = \sum_{j=1}^{4} X_i + Q_r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (71)$$

$$D_m + X_8 - X_6 - X_7 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (72)$$

$$D_i - X_8 - X_9 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (73)$$

and,

$$Q_i = \phi(D_d, D_m, D_i) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (74)$$

where Q_i is the discharge of urban effluent, mgd.

Objective Function

Employing the minimum cost criterion described in Section II, and the cost functions described in the previous paragraphs, the objective function can be formulated. The objective function consists of three basic segments; (1) cost of water supplies available at the raw water treatment intake; (2) the costs of raw water treatment; and (3) the costs of recycling wastewater directly to the municipal and urban demands.

If P_1 is defined as the water supply cost, then

$$P_1 = \sum_{j=1}^{4} c_j X_j + (Q_r - X_7 - X_9) c_r \quad . \; . \; . \; . \; . \; . \qquad (75)$$

in which c_j is the unit costs of the j^{th} water source, X_j is the quantity selected from the j^{th} source, and c_r is the costs of reused water, derived from the regression with values of TDS in the flows. The polynomial expression for c_r will be in the following form,

$$c_r = A_1 + A_2 C_{tr} + A_3 C_{tr}^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \qquad (76)$$

where A_i are the regression coefficients. The variable C_{tr} is the coordinating link between the urban wastewater reclamation system and the urban water supply and distribution system.

The Illinois State Water Survey (1968) indicated that in terms of 1964 dollar value, the capital construction costs for raw water treatment facilities, including screening, flocculation, clarification, rapid sand filtration, and chlorination, could be expressed as:

$$Y = 0.323 \, Z^{0.65} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \qquad (77)$$

In addition, this source also showed the operation and maintenance costs could be determined by:

$$y = 1.21\ Z^{-0.35} \qquad (78)$$

where y is the costs of the treated water in $/1000 gal and Z is the capacity of the treatment facilities in mgd. If these costs are evaluated at their present-worth, added to the interest costs, and extended in a uniform series over a design life of 30 years with a discount rate of 5%, then the total annual costs, in terms of 1964 dollars, can be written as,

$$P_2 = 200\ Z^{0.74} \qquad (79)$$

where P_2 represents the total annual costs in dollars per million gallons treated.

The final component of the objective function is the costs of the recycled water which is sent directly to municipal and industrial demands. From the preceding definitions, P_3 can be representative of the costs and written as,

$$P_3 = (X_7 + X_9)c_r \qquad (80)$$

or more completely:

$$P_3 = (X_7 + X_9)(A_1 + A_2C_{tr} + A_3C_{tr}^2) \qquad (81)$$

The complete objective function can now be expressed:

$$y = \min_i \sum_{i=1}^{3} P_i \qquad (82)$$

These previous functions do not include several of the costs encountered in supplying water to urban demands. For example, the costs of the distribution system, pumping, and storage are not included since they are assumed to be common to all aspects of the model. In addition, the functions were developed on the basis of differing dollar values which can be corrected by applying an adjustment factor taken from published cost indexes.

SECTION VI

COORDINATION OF AGRICULTURAL AND URBAN WATER QUALITY MANAGEMENT

Introduction

The interrelated and competing uses of water often result in complex problems in areas where water resources are inadequate to meet the needs of potential interests. The seriousness of these problems depends on the viewpoint of the observer and efforts are necessary to arbitrate the differences between those responsible for the problems and those adversely affected. One of the most difficult of these problems to resolve because of its widespread occurrance is water quality degradation. The distinction between water supplies and wastewater return flows is slight since many instances exist in which one individual's effluent is another's supply. The ultimate objective is therefore to maximize the utility of the resource on a regional scale.

In order to insure equitable allocation of a resource extending beyond local boundaries, the administrative structure of the water right was formulated. Along with the concept of a water right, other institutional factors have been implemented to control the flow and use of water. Although such factors promoted optimal water use when first implemented, no efficient means of updating these criteria for later water resource allocation were included. The institutional structures constructed in one era may thus become barriers at a later date to optimization of water resource use. When such conditions are encountered, investigation into the feasibility of alternative courses of action is initiated.

In arid and urbanizing areas like the Utah Lake region, the best policy for water quality management usually lies between the alternatives of complete emphasis on urban pollution or on agricultural pollution. Water quality management programs, instituted to insure the continued use of the resource elsewhere, must consider which combination of treatments is the most efficient, (e.g., the strategy with the greatest cost-effectiveness). As a result, an important question to be addressed is the optimal policies of coordinated water quality management between agricultural and urban users. One method which is commonly employed for evaluating such decisions is to model the water use system at a level of detail that will yield the solutions to the problem.

This section is presented to discuss the formulation and application of a model designed to facilitate this type of investigation. In the Utah Lake area, four major divisions in the region surrounding the lake have been made (Huntzinger, 1971): (1) American Fork-Lehi District; (2) Provo District; (3) Spanish Fork District; and (4) Elberta-Goshen District. The bulk of both agricultural and urban demands in the basin are contained within these districts. Consequently, the model developed in this section is intended to represent the water quality management decisions on this district level.

Model Description

The coordination of agricultural and urban water quality controls is a necessary requirement for such pollutants as total dissolved solids (TDS), often referred to as salinity. However, the costs of urban salinity control were shown in SECTION IV to be somewhat dependent on reductions in Biochemical Oxygen Demand (BOD), as well.

The urban wastewater treatment segment of this model is equivalent to the model described in SECTION IV when the levels of reuse are zero. Relying on that previous discussion to sufficiently acquaint the reader, the emphasis in the following paragraphs will be on the agricultural segment of the model.

A management level model of an area consisting of both agricultural and urban water uses can be conceptually organized as shown in Figure 14. Such a model assumes that diversions to the sectors are independent in nature and thus does not allow for the subsequent use of effluents by another use. This type of operation is prevalant in the Utah Lake study area, but can be modified easily to represent the more general situation.

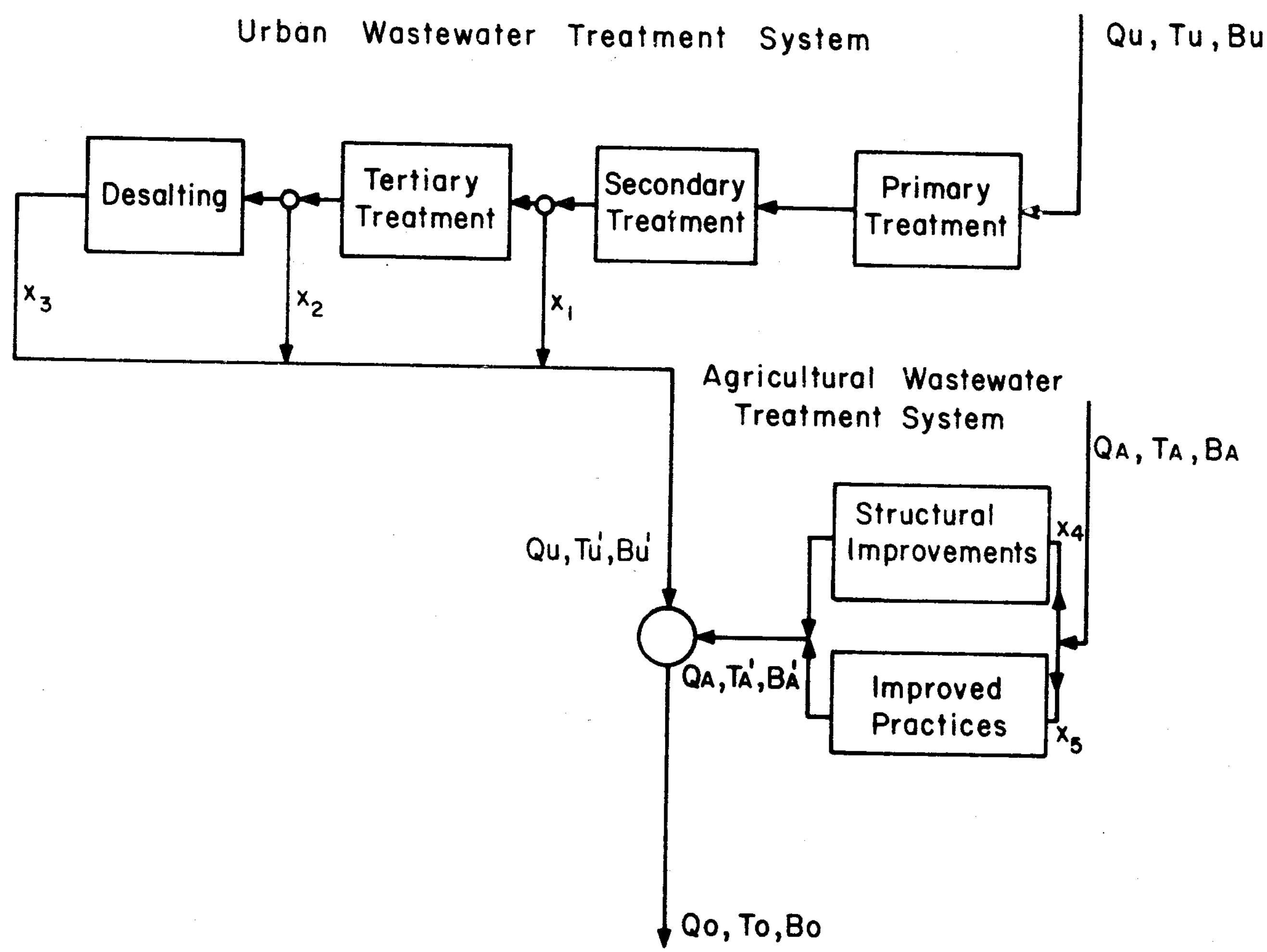

Figure 14. Utah Lake district model.

The agricultural segment of the model is divided into two treatments for reducing TDS concentration in the return flows: (1) improving the water use practices in the districts; and (2) rehabilitating the structural elements of the system. In the following paragraphs, these two measures will be described in detail.

Improved Practices

Water diverted for agricultural purposes in the western United States is used primarily for irrigation of cropland to supplement insufficient natural precipitation. The individual network of flows in the agricultural area can be broadly divided into a groundwater subsystem, a root zone subsystem, and a surface subsystem, each of which are intricately interrelated with one another.

The surface subsystem begins with the diversion of water from rivers, streams, or reservoirs into a conveyance system consisting of either canals, ditches, or conduits. From these conveyance networks, water is transmitted to the areas of cropland to be irrigated, but in route some of the flows are lost by seepage into the groundwater subsystem and evaporation to the atmosphere. The remainder of the diversions are applied to the croplands and enter the root zone subsystem. Some of the water applied to the cropland, however, may result in field tailwater which then returns to the natural surface system for conveyance downstream.

Flows entering the root zone subsystem by way of the soil surface are usually the largest percentage of the total diversions. From here, plants draw water through their roots up into the leaves where it transpires into the atmosphere. In addition, some moisture may evaporate directly from exposed soil surfaces or percolate into the groundwater.

Finally, water from seepage, percolation, subsurface inflows from adjacent areas, and water naturally occurring in the area are collected in the groundwater subsystem. In most areas, the storage capacity in this subsystem is extensive, which is useful in several ways as noted in SECTION V. Flows in the groundwater basin are partially interactive with the other subsystems via drainage interception and capillary water rise.

Associated with each of the components in the water flow system is a TDS concentration. These concentrations are affected by the concentrating effects of evaporation and the loading effects derived from the contact with dissolvable materials. If a particular segment in the flow network is especially detrimental with regards to these quality criteria, then practices which introduce excessive water into such areas should be alleviated, or improved.

The effective treatment of conditions which concentrate salts is generally a matter of correcting the effects of poor water management practices. For example, the eradication of phreatophytes and the minimization of water surface areas by eliminating unnecessary ponds and marshes, are often feasible methods of salinity control. Utah Lake is the largest concentrator of salts in the study area with the salt concentration being more than doubled. As a result, lake diking to reduce the surface area of Utah Lake is currently under serious consideration as part of the Bureau of Reclamation's Central Utah Project (Skogerboe and Huntzinger, 1972).

The contact of deep percolating water with soils and groundwater aquifers results in a leaching of salts from these materials, or the dissolving of mineral constituents which are taken into solution. This salt loading effect, often called "salt pickup," is observable in most areas, but the rate of salt pickup varies widely depending upon geologic characteristics and the previous effects of irrigation. Salt pickup is generally minimized when efficient irrigation practices are used. Total control is usually not possible since some leaching is necessary to maintain a salt balance in the root zone region of the soil to insure agricultural productivity.

The reduction of TDS concentrations in irrigation return flows can be accomplished by improving the efficiency of the irrigation practices. The term "irrigation efficiency" has been defined differently throughout the technical literature, but is intended to denote the percentage of the total agricultural diversions that is beneficially used. To increase this efficiency, radical departures from existing practices may be necessary. For example, tighter control of water in the conveyance systems reduces seepage and dumping of surplus flows into natural waste-ways, both of which cause salinity problems. In addition, different irrigation methods may achieve higher levels of application efficiency, which is the percentage of water applied to the cropland that is consumptively used by the plants. These improved methods control salinity by reducing the quantities of deep percolation losses.

Since significant variations in irrigation efficiencies have been shown among individual farmers, improvements in irrigation practices can be made in such a manner as to correct the most inefficient first. This assumption yields cost functions which have the characteristic of increasing marginal costs with the percentage of the area treated. The relationship derived for this study is shown in Figure 15, along with a similar curve for the structural improvements to be described next. These cost-effectiveness functions were generated from previous research experience in the area, as well as feasibility studies conducted to evaluate the potential for such improvements.

Structural Improvements

Many methods and structures which have been developed for irrigating land provide considerable flexibility in selecting the proper system for individual farm needs. In these systems, the following functions should be

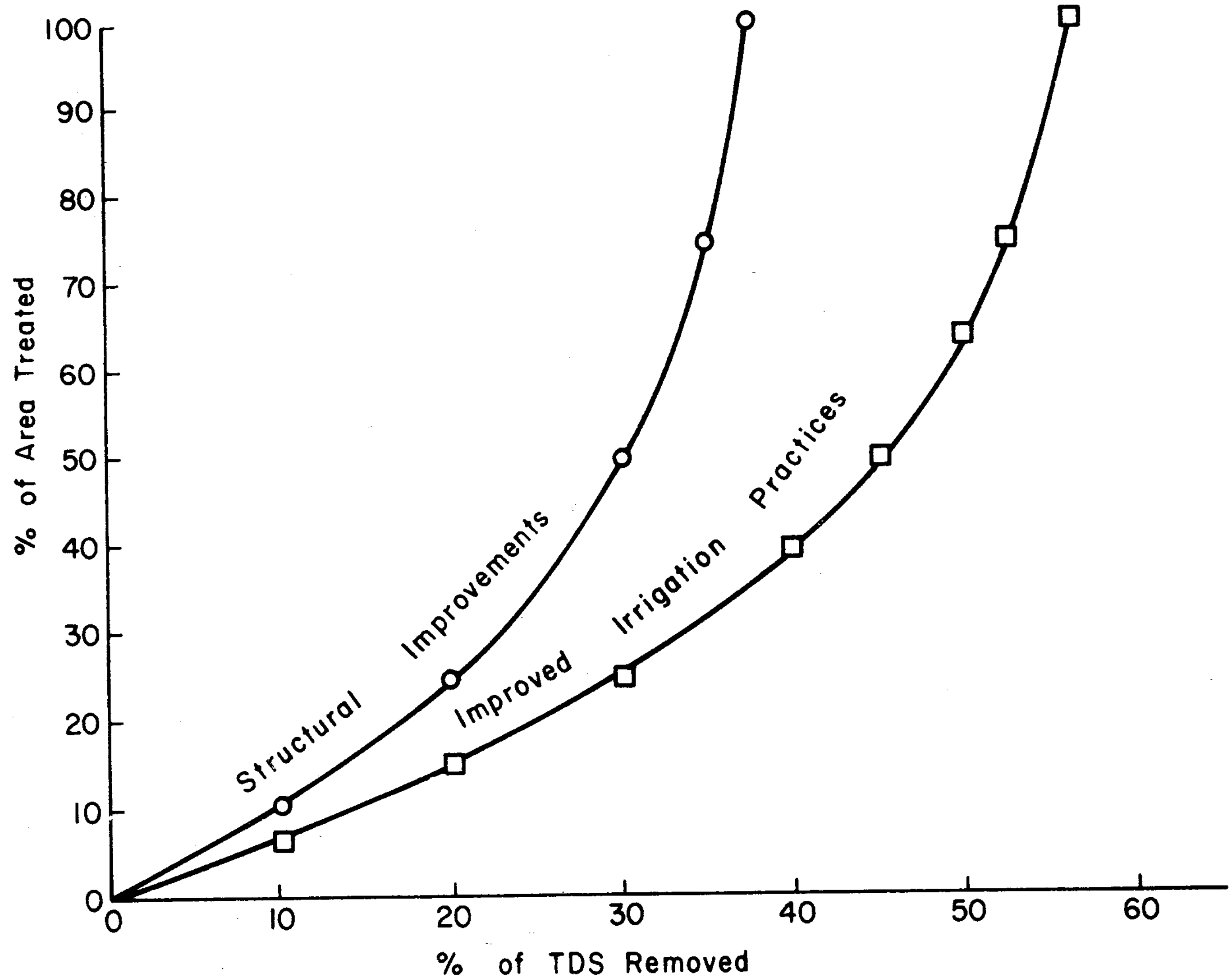

Figure 15. Cost-effectiveness relationships for TDS reductions in agricultural return flows in the Utah Lake drainage area.

efficiently employed: (1) water should be delivered to the fields when needed and in amounts necessary to meet the crop demands; (2) complete control of the water should be provided and accurate measuring techniques practiced; (3) water should be distributed evenly into the soil of each field with adequate disposal of wastewater or the capability of reuse if desired; and (4) allow easy movement of farm and maintenance machinery. When any of these categories are not effectively fulfilled by the system, water quality deterioration can be expected from excession seepage, deep percolation, poor drainage, phreatophytes, and high evaporation losses.

At the present, many irrigation systems are in poor condition because little or no need has existed for improved water management. With few exceptions, water supplies for agricultural lands have been abundant, thus leading to low costs for water, and excessive uses. These conditions have resulted in a significant deterioration in water quality in many areas of the western United States.

The structural costs to alleviate these conditions, which are shown in Figure 15, also exhibit increasing marginal costs with increasing scale. However, the effectiveness of structural improvements is not expected to be as great as the effectiveness of improved irrigation practices, but structural improvements must be implemented in order to achieve higher levels of irrigation efficiency.

Model Formulation

In order to operate the model in a manner which evaluates alternative water quality control decisions, it is necessary to formulate the mathematical description in an optimizational format. Optimization requires three essential parts: (1) a criterion upon which alternatives are ranked; (2) an objective function relating the criterion to the alternatives; and (3) an array of constraints which limits the search for the best alternative to a specified feasible region. The model constraints are not always necessary if the search is assumed to be unbounded. Since this is not a general condition, constraints are usually considered as part of the model.

The optimization criterion has been selected and discussed in SECTION II, where cost minimization was shown to be the most realistic parameter for this study. In the following segments, the discussion will emphasize first the objective function and then the model constraints.

Objective Function

Beginning first with the urban sector of the model, it may be worth repeating the functions derived in SECTION IV. Recalling Equation 34, the cost functions have the typical form,

$$Y = aZ^m \qquad (34a)$$

where Y is the cost in millions of dollars annually, a is a coefficient, Z is the capacity of the facility in millions of gallons per day (mgd), and m is an exponent usually less than 1.0. The costs of construction, as well as operation and maintenance, were then outlined for each of the treatment operations; namely, primary, secondary, tertiary, and desalting. A summary of these will be omitted here; but let P_1, P_2, P_3, and P_4 be the variable unit costs for primary, secondary, tertiary, and desalting plants, respectively. Referring to Figure 14, the annual costs of urban wastewater treatment in a particular aggregated district can be written,

$$Y_u = Q_u(P_1+P_2) + (X_2+X_3)P_3 + X_3P_4 \quad . \quad . \quad . \quad . \quad . \quad (83)$$

in which Y_u is the total annual costs in \$ million, Q_u is the average daily outflow from the urban sector in mgd, and X_1, X_2, X_3 are the mixing flows released from each phase of the treatment to meet a specified level of water quality in the effluents. In this model, it is assumed that tertiary treatment is necessary for all flows to be desalted, and BOD removal efficiencies in this model have been fixed at 85% for the combined primary-secondary treatment and 99% for the tertiary treatment facility, while the TDS removal efficiency of the electrodialysis desalting plant is 90%. Consequently, to achieve any particular value of either TDS or BOD concentration in the effluents, only the capacities are changed.

The agricultural water quality management costs are quite different than the urban costs. In this sector of the model, the salts which can be removed are only those actually being picked up in the area. In addition, a small reduction in TDS concentrations can be achieved by minimizing phreatophytes and evaporation from open water surfaces. As a result, a minimum TDS concentration in agricultural return flows, accomplished when 100% of the district has been structurally rehabilitated and modified to exhibit the most effective irrigation practices, is a function only of the portion of the salts in the flows added by the district. It may be worth noting in Figure 15 that if all the area is treated, the maximum effect is approximately 90%. This is to account for the small fraction of the problem which must be left unresolved because of the leaching requirements in the root zone.

For any intermediate reduction in TDS concentrations in agricultural return flows, it is necessary to select the optimal balance between structural and practice improvements. Consequently, in each optimization between the agricultural and urban sectors, a sub-optimization is necessary in the agricultural sector. To formulate this problem, it is necessary to define the total annual structural cost in a district as C_s and the total annual practice improvement cost as C_p. Then let X_4 and X_5 be the fraction of the expected TDS reduction accomplished by each alternative measure. Thus, if a level of TDS of T_A' is desired in the return flows, the costs of doing so can be written,

$$Y_a = C_s\phi_s(X_4) + C_p\phi_p(X_5) \quad \ldots \ldots \ldots \quad (84)$$

in which Y_a is the total annual costs in \$ million, and ϕ_s and ϕ_p are the respective functions in Figure 15. If the percentage of salts to be removed is defined as,

$$CT = 1.0 - \left[\frac{T'_A - T_{min}}{T_A - T_{min}}\right] \quad . . . . \quad (85)$$

where CT is the fraction removed, T'_A is the concentration desired in the return flows, T_{min} is the minimum achievable in the district, and T_A is the initial concentrations in the effluents, then;

$$CT = X_4 + X_5 \quad . . . . \quad (86)$$

The optimal choice of values for X_4 and X_5 is thus determined by minimizing Equation 84, subject to the constraints expressed in Equations 85 and 86.

Once the minimum costs for the urban wastewater treatment, Y_u, have been determined and also the agricultural costs, Y_a, the objective function for each district can be written;

$$Y = \min \left[Y_u + Y_a\right] \quad . . . . \quad (87)$$

The optimization thus attempts to find the best policy for allocating water quality management responsibilites between the agricultural and urban water user. This solution is achieved in two levels; by first optimizing each individual sector and then secondly, optimization of the two sectors together.

Model Constraints

In order to insure that the solution to this problem is both realistic and feasible, it is necessary to impose constraints on the model. Again, beginning with the urban wastewater treatment system, assume that a TDS constraint on the district effluent has been set at T_o, and further assume that the corresponding BOD constraint is set at B_o. If by some means it was previously determined that the optimal level of urban effluent TDS and BOD concentrations were T'_u and B'_u, then this sector's constraints could be written as,

$$\frac{X_1(0.15 \cdot B_u) + (X_2 + X_3)(0.01)(0.15 \cdot B_u)}{Q_u} \leq B'_u \quad . \; . . \qquad (88)$$

and

$$\frac{(X_1 + X_2)T_u + X_3(0.1)(T_u)}{Q_u} \leq T'_u \quad . \; . \; . \; . \; . \; . \qquad (89)$$

where Q_u, T_u, and B_u are the initial effluent flows, TDS, and BOD, respectively; and

$$X_1 + X_2 + X_3 = Q_u \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \qquad (90)$$

In a similar manner, if the predetermined optimal water quality characteristics for the agricultural sector were T'_A and B'_A, then these constraints could be written as indicated previously in Equations 86 and 87.

Since the variables T'_u, B'_u, T'_A, and B'_A are in reality the main interest in this model, then an overall set of constraints must be imposed; namely,

$$\frac{Q_u B'_u + Q_A B'_A}{Q_o} \leq B_o \quad \text{. . . . .} \quad (91)$$

and,

$$\frac{Q_u T'_u + Q_A T'_A}{Q_o} \leq T_o \quad \text{. . . . .} \quad (92)$$

where:

$$Q_u + Q_A \leq Q_o \quad \text{. . . . .} \quad (93)$$

SECTION VII

OPTIMIZING REGIONAL WATER QUALITY MANAGEMENT STRATEGIES

Introduction

During recent decades, improvements in hydraulic design, equipment, and construction methods have increased the feasibility of transporting large quantities of water. For this reason, the development and transfer of water resources in areas with abundant supplies to areas encountering shortages has become widely practiced. Rapid growth and concentration of urban demands have often been met with interbasin transfers because of their high quality and economic feasibility. Other alternatives, such as the acquisition and transfer of agricultural water rights, have been used to a lesser degree because of the complex and often discouraging legal procedures. In addition, the reclamation and reuse of wastewater has previously been prohibitive because of high costs.

The decision to import water in each instance has been a local rather than a regional optimum. These diversions are not without some inherent disadvantages, one of which is of primary concern in this section; namely, water quality deterioration. As a consequence of these limited considerations, at least two major water quality problems are manifested. First, the export of good quality water from a watershed reduces the dilution capacity of that system to assimilate and transport the waste materials occurring therein. This condition is a major argument against interbasin water transfers, since they tend to restrict economic growth in the basins of origin. The second problem is felt downstream from the local area when the effect of the effluents overwhelms the existing assimilative capacities in the remaining flows. When this occurs, the downstream uses are restricted to the more insensitive applications, thereby again being economically penalized. The resolution of the first problem is yet to be evaluated, but the second condition is of particular interest in this study.

The purpose of this section is to formulate a model of the basin wide water quality management decisions for areas involving both urban and agricultural water quality problems. In the previous section, the model presented represented individual subsystems in a region and was shown to be relatively general in nature. The model to be formulated in this section was designed specifically for the Utah Lake drainage area and is therefore less applicable to other areas. The basic principles of the model are general, however, so that application to other problems can be facilitated by modifying the geometry of this model to fit the characteristics encountered in another area.

Model Description

Some of the flows within the Utah Lake drainage area are used for municipal and industrial demands in Utah Valley and the Salt Lake City metropolitan area. These uses require careful examination of a number of water quality parameters, but the most important is probably the concentrations of TDS. Municipal and industrial water supplies presently derived from the area have excellent quality, while the return flows from Utah Valley, which are collected in Utah Lake, create a salinity problem; however, there is not a significant BOD problem in Utah Lake at the present time. Thus, TDS was selected in this model to indicate the effects of alternative water management strategies in the region.

The model proposed in this section is subdivided into three major measures of salinity control. The first is the management of salts in the return flows from the lands surrounding Utah Lake. In SECTION VI, these inflows were denoted as district effluents and consequently this alternative can be defined as district management. The reduction of lake evaporation is the second salinity control measure investigated in the model. Finally, exposing a fraction of the flows leaving the region (Jordan River) to desalination is considered. When these alternatives are coordinated in a management level model, such as shown in Figure 16, questions concerning optimal water quality

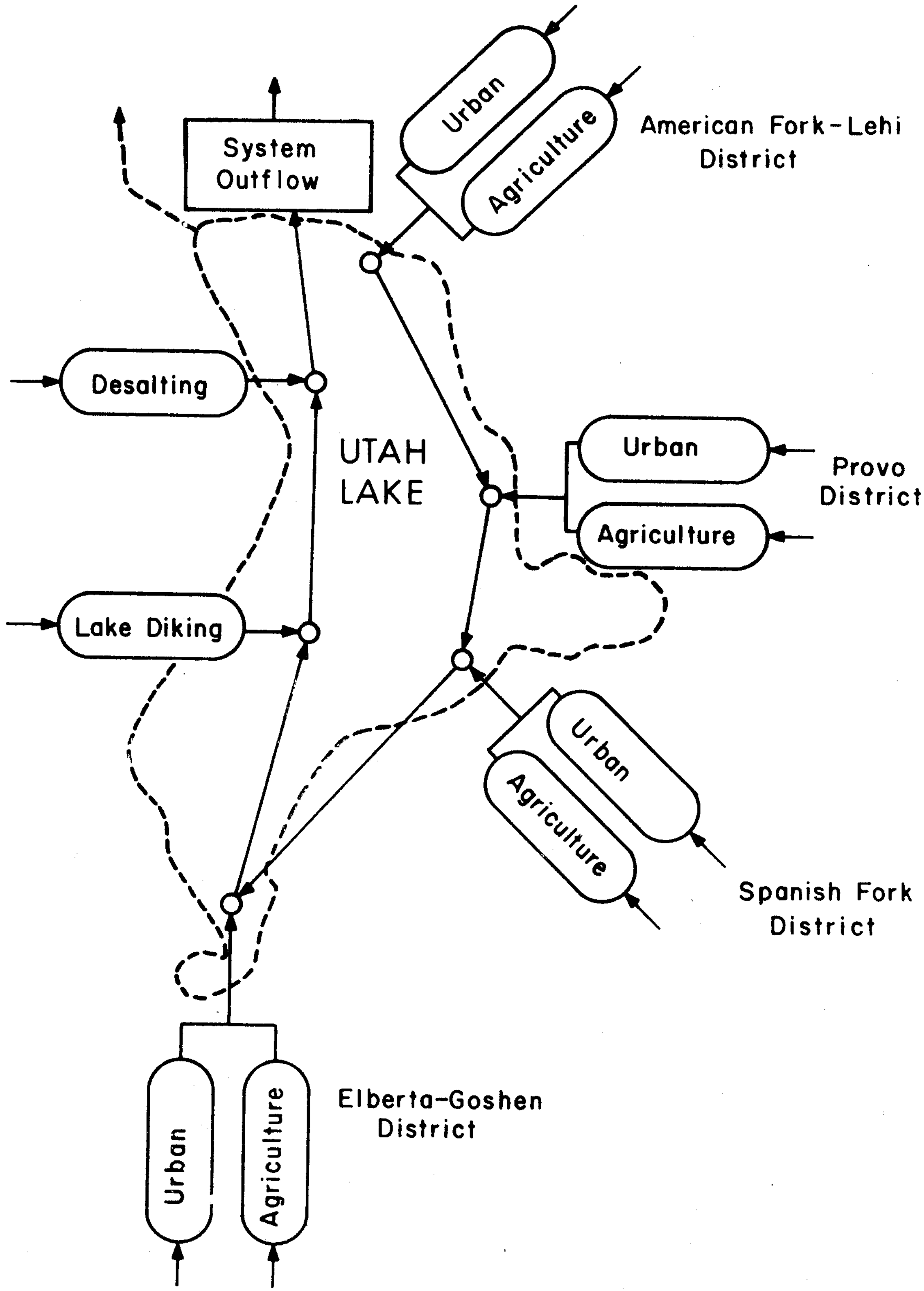

Figure 16. Utah Lake area water quality management model.

management can be addressed. A more detailed description of these model components, which is presented in the following paragraphs, is helpful in developing an understanding of the system.

District Water Quality Management

In the previous section of this report, a district model was presented which describes the interaction of urban and agricultural water quality management. At the next level of investigation, it is interesting to determine the optimal strategy for controlling water pollution by coordinated operation among the districts. To accomplish this task, the district models can be implemented into an overall search for such an optimum. The technique used in this segment of the model is a dynamic programming philosophy.

In previous studies, the major districts were called collectively Utah Valley (Hyatt, Skogerboe, Haws, and Austin, 1967, and Huntzinger, 1971). Following this lead, the segment of the regional model describing the Utah Valley area operates by finding the minimum cost policy for achieving a predetermined aggregate TDS concentration in the flows entering the lake.

Lake Diking

As the Utah Lake evaporates approximately one-half of the flows which enter it, the concentrations of salts are correspondingly doubled. (Hyatt, Skogerboe, Haws, and Austin, 1967). As a result of this action, the effect of the lake far surpasses any other single cause of salinity in the area. This realization has been incorporated into the Bureau of Reclamation's massive Central Utah Project. Dikes are planned to separate both Goshen and Provo Bays from the lake. From the estimated costs of the dikes, a function with increasing marginal costs with scale was derived, which will be presented in a following report. This function allows the model to evaluate various diking alternatives and test their feasibility in light of the objective of regional water pollution control.

Desalting

A great deal of research has been undertaken to determine the feasibility of reclaiming and reusing water which was otherwise unfit for use because of poor water quality. Among the pollutants of most concern, the inorganic salts are the most significant. Investigations involving the application of desalting technology have been undertaken in numerous areas. In 1968, the Utah Division of Natural Resources, as part of the long range state water plan, joined with the Office of Saline Water of the Department of the Interior and the U.S. Atomic Energy Commission to test the feasibility of a multipurpose desalting, steam production, and electric power generation facility in the Utah Lake area which would supply some of the metropolitan needs of Utah (Haycock, Shiozawa, and Roberts, 1968). The recommendations were generally positive regarding desalting, stating that ". . . desalting should be considered a potential source of water supply to meet the State's growing water needs."

The scope of this model does not consider desalting as a "source" of water, but rather an amendment to make existing resources usable. This is admittedly a fine distinction to make. However, the view taken here is one of water quality management in the total resources of a large drainage basin.

The alternative methods suggested by the Utah study center primarily on electrodialysis, which is also utilized in this model. This, of course, simplified much of the modeling, since this method has received close scrutiny in a prior section of this report.

Model Formulation

The time interval used in this model was set as one year in this study. The data requirements are thus annual averages expressed on a daily basis. The scope of the model is similar to those previously developed in that it is in an optimization structure. Consequently, the major divisions in this phase of the section can be divided into a discussion of the objective function, which compares alternatives, and the constraints which control the search for the optimal solution.

Model Objective Functions

The cost of achieving a specified reduction in the TDS levels in the area outflow consists of the costs attributable to management in Utah Valley, lake diking, and desalting. The first consideration in this serial system is the Utah Valley segment. Assume initially that the TDS concentration in the valley effluents under a predetermined policy are known and defined as C_{in}. In order to accomplish this objective, it would be necessary for each district effluent to be treated sufficiently to make the net effect equal to the standard. This, of course, requires an input to each area according to an optimal policy. It is this allocation of costs among the districts that is of interest.

Suppose from the district model previously formulated, the minimum costs for achieving a TDS level in the i^{th} district were represented by P_i. Then, the costs of water quality management in all Utah Valley would be the sum of the costs in the districts,

$$Y_v = \sum^{i} P_i \quad . . . . \quad (94)$$

where Y_v is the total annual costs.

The next aspect in the objective function formulation is to link the diking alternative with treatments in Utah Valley. Let Q_o be defined as the daily average outflow from the lake, thus

$$Q_o = \sum^{i} Q_i - Q_l \quad . . . . \quad (95)$$

in which Q_i is the inflow from the i^{th} district in mgd; and Q_l is the average daily lake surface net evaporation, in mgd. The concentration of TDS entering the lake, C_{in}, can be written in terms of these variables as:

$$C_{in} = \frac{\overset{i}{\Sigma}\, Q_i C_i}{\overset{i}{\Sigma}\, Q_i} = \frac{\overset{i}{\Sigma}\, Q_i C_i}{Q_o + Q_l} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \qquad (96)$$

The concentration of TDS in the lake outflow can similarily be written,

$$C_o = \frac{\overset{i}{\Sigma}\, Q_i C_i}{Q_o} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \qquad (97)$$

If Equation 96 is subtracted from Equation 97, the relationship between the inflow concentration and that for the outflow is determined,

$$C_o - C_{in} = \overset{i}{\Sigma}\, Q_i C_i \left[\frac{1}{Q_o + Q_l} - \frac{1}{Q_o}\right] \quad . \; . \; . \; . \; . \qquad (98)$$

which reduces to:

$$C_o = \frac{C_{in} \overset{i}{\Sigma}\, Q_i}{Q_o} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \qquad (99)$$

This expression can then be used to evaluate the costs of lake diking as a function of salinity reductions. To be consistant, however, it is helpful to express the costs of the lake diking measure in terms of a percent reduction. As a result, it is possible to define this fraction as,

$$n = \frac{C_o - C_{in}}{C_p - C_{in}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \qquad (100)$$

where n is the decimal fraction and C_p is present concentrations. The cost-effectiveness function for lake diking is similar in shape to the curves of Figure 15. Thus, the costs of lake diking can be written as,

$$Y_l = a \left[n\right]^m \quad \ldots\ldots\ldots\ldots \quad (101)$$

in which a and m are the functional constants.

Finally, the costs of the desalting alternative can be added to the objective function. Assuming the TDS removal efficiency is fixed, the degree of desalting necessary for the outflows to meet any specified standard can be calculated. If the final effluent quality is defined as C_o'', a simple dilution equation reveals that:

$$Q_o' = \frac{Q_o C_o'' - Q_o'' C_o}{C_o'} \quad \ldots\ldots\ldots\ldots \quad (102)$$

where Q_o' is the desalting plant capacity in mgd, C_o' is the quality of the desalted flows in mg/l, and Q_o'' is the flows passing by the desalting plant in mgd. Thus, the costs of the desalting can be determined as illustrated in SECTION IV, which can be denoted by Y_d.

By properly selecting the amount of salts to be removed in the districts according to an optimal allocation among the districts, the degree of lake diking, and the desalting capacity, a minimum cost can be computed for accomplishing an effluent standard, C_o''. Thus, the objective function is:

$$Y = \min \left[Y_u + Y_l + Y_d\right] \quad \ldots\ldots\ldots\ldots \quad (103)$$

Model Constraints

Beginning with the Utah Valley phase of the model, assume that the degree of treatment for the entire valley has been set at a TDS concentration of C_{in} as noted before. Then, a water quality constraint on the valley must be written to insure this goal is reached:

$$\frac{\sum^{i} Q_i C_i}{\sum^{i} Q_i} \leq C_{in} \qquad (104)$$

A constraint is also necessary in order to achieve coordination of the three types of salinity control measures taken together. This function can be written as the solution for C_o'' in Equation 102:

$$C_o'' = \frac{Q_o' C_o' + Q_o'' C_o}{Q_o} \qquad (102a)$$

These constraints link the model proposed in this section together and allow the model to optimize the water quality management strategies for the area.

Model Operation

This model operates in four basic levels:

(1) Optimization of urban and agricultural wastewater treatment scheme on an individual basis as outlined in the previous section;

(2) Optimal coordination of urban and agricultural salinity control on a district scale, which was also discussed in the previous section;

(3) Optimization of water quality strategies among districts; and

(4) Optimization of salinity control including Utah Valley, Utah Lake diking, and desalting.

Because the model involves four levels of optimization, it is helpful to explain the model operation in detail in order to understand the evaluation presented in this section.

Suppose the concentration of dissolved salts in the outflows from the Utah Lake area were to be reduced by 100 mg/l and it was of interest to know the least cost policy for achieving this objective. The first step would consist of making an intelligent "guess." The first feasible solution would thus specify the reductions in TDS concentrations necessary in the return flows from Utah Valley, the amount of diking to be constructed, and the capacity of the regional desalting plant. The total costs, as well as the marginal costs (constrained derivatives), for both desalting and lake diking can be immediately calculated since they are represented by single-cost functions. The costs for the Utah Valley segment, however, must be determined.

With the value of the composite TDS concentration for the return flows from Utah Valley specified, the next step is to assume an initial distribution of salinity reductions throughout the districts. Having accomplished this step, the model developed in the previous section is used to optimize the urban and agricultural water quality control within the districts. Then, these solutions are used to calculate the constrained derivatives from which the solution can be tested for a minimum. If such is not the case, the allocation among districts is modified to reflect another solution which is close to the minimum. This procedure is repeated until the salinity control in Utah Valley is represented by a minimum cost solution.

At this point, the marginal costs for the Utah Valley segment is compared to those of the other two salinity measures to check for a model optimum. If the solution is not optimal, another solution is computed and this entire process is repeated. When the minimum cost solution for the entire Utah Lake drainage area is found, a complete and comprehensive strategy for salinity control has been outlined.

SECTION VIII

SUMMARY AND CONCLUSIONS

Introduction

Water management in arid urbanizing regions is a problem requiring careful decision making supported by extensive investigation into all alternative actions having particular merit. The most commonly employed tool for accomplishing such comprehensive studies is the formulation of mathematical models of the water use systems in these regions. Such models are useful in not only determining optimal courses of action to meet the changing needs, but also produce valuable information concerning the operational aspects of large and complex water use systems.

This report is the first in a series addressing the questions of water management in metropolitan areas and their surrounding watersheds. In this writing, the model formulations which accomplish these objectives are presented, with this section summarizing the results of the modeling developments.

Summary

As a means of testing the value of the models developed in this report, two case studies will be presented in subsequent reports. The first area selected for study was the Denver metropolitan area, which is characteristic of arid urban centers facing critical water shortages and increasingly stringent controls on the area's effluent water quality. The Utah Lake drainage area in central Utah was chosen as an area where a large scale regional water quality study could be made. In this area, downstream water uses are severely affected by high TDS levels resulting from salts acquired in the Utah Valley (both agricultural and municipal) surrounding the Utah Lake, as well as the concentrating effect of evaporation from the lake. Together, these two areas typify water quality management problems often encountered in the western United States.

The comparison of alternative strategies for water management implicitly assumes an optimizational format. As a result, the initial sections of this report describe the optimization criterion employed in the study and the principal optimization technique developed. A minimum cost criterion was selected from among the available economic indicators because of the generally restricted nature of water resources. State systems for allocation of water resources and federal policies for water quality control tend to isolate individual water uses from each other at the management level. Consequently, much of the decision processes in these areas center around meeting the demand and water quality restrictions at minimum public cost.

The technique developed for minimizing costs in this study is a general differential algorithm derived from the basic mathematical approach to the derivation that added generality to the method, thereby making it applicable to problems which may be linear, quadratic, or geometric in nature, as well as non-integer non-linearities. The method is also general in that it allows linear or non-linear constraint functions of either the equality or mixed inequality type.

Four individual mathematical descriptions were prepared for different aspects of regional water quality management. The first application was made to the typical urban wastewater collection and treatment system. A model consisting of the essential elements of primary, secondary, tertiary, and desalting treatments were formulated together in a mixing model. The costs of the facilities, as well as operation and maintenance costs, are minimized subject to the criteria established for the effluents. Water which is to be recycled is priced by computing unit differences between the optimized system costs with no reuse and those with a specified level of recycled diversions. Each of the wastewater treatment costs exhibit decreasing marginal costs with scale, thereby encouraging system consolidation.

The remaining two aspects of urban water management; namely water supply and distribution, were next modeled and then coupled with the wastewater treatment model via reuse to represent the complete urban water system. The alternatives for water supply include stream flows, interbasin transfers, agricultural water right acquisition and transfer, groundwater, and of course, reused wastewater. The costs of these supplies were assumed to be linear, making the decisions in the model primarily functions of the effluent standards on the wastewater, since the relative feasibility of the recycled flows was the basic variable in this regard. From the sources of water, the flows proceed through the raw water treatment to the individual urban demands. In this model, a distinction was made in these uses on the basis of both a priority for quantity and quality. Domestic uses which are typically defined as household demands, as well as those demands which are most sensitive to water quality, were delineated. In addition, the municipal demands consisting of vegetative needs, fire protection, and other uses were also delineated; as were the host of industrial type needs. The model allows recycling to either be accomplished through the existing raw water facilities or directly to both municipal and industrial demands. This characteristic provides a great deal of flexibility in evaluating alternative policies in the urban system.

These previously noted models were designed specifically for the urban area. Also of interest in this study was the interaction of agricultural and urban water quality problems. To study this question, a model was developed to describe first individual agricultural and urban wastewater treatment functions and secondly to coordinate them together as a means of optimizing water quality management from a locality. The model (termed a district model) is general in nature and is applicable to similar problems in other areas.

Finally, the district level model was incorporated into a regional or basinwide water quality management model of the Utah Lake drainage area. This model also consisted of desalting and lake diking measures for evaluating salinity control strategy for the area. By optimizing the levels of salinity control responsibility among the various alternatives, a comprehensive strategy is generated for water quality management.

These models are multidimensional descriptions of the basic water management strategies in urbanizing areas. They answer questions of concern for planners and officials of local, state, and national government by evaluating management policies. The scope of these models is, however, most useful in testing the effects of the legal, social, and political structures which have emerged to regulate and allocate water resources.

Conclusions

The primary conclusions developed from this portion of the study involve mainly the philosophy of modeling. The water resources system is already complex without adding the immense details of an urban structure. Therefore, a temptation exists to create models which are detailed and exhaustive. The authors feel that such a direction does not facilitate decision-making, but rather clouds the issues.

Because of the interrelated nature of water management decisions, it is more feasible to limit models to either of two purposes. First, it is helpful to provide models to planners and operators of water systems which aid their decision making. Such models must be designed to answer specific questions of interest and should be as simple as possible. The second aspect deals with questions involving the integration of several levels of decision-making. These models should be composed of simple unipurpose sub-models coordinated by a more sophisticated modeling philosophy. The advantages of this stance is obvious. Each segment of the complex system is best known to its operators. Thus, a model of these components can be more realistically detailed, thereby resulting in more representative responses.

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