MODLES DESIGNED TO EFFICIENTLY ALLOCATE IRRIGATION WATER USE BASED ON CROP RESPONSE TO SOIL MOISTURE STRESS

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INTRODUCTION

This report examines models designed to allocate limited irrigation water to crops throughout the growing season so as to obtain the optimum return from water applied. This is a complex problem involving a great many variables including plant growth over the season, soil moisture status and weather conditions that affect evapotranspiration.

Several models have been developed to estimate the yield effects of various levels of soil moisture available to irrigated crops during the growing season. A number of simulation and linear programming models have been developed to project net returns from various alternative irrigation regimes, ranging from single crops to entire farms or irrigation systems. Central to these models is crop response to situations of soil moisture stress at various periods throughout the growing season. The effect of soil moisture stress on crop yield has long intrigued plant physiologists, agronomists, farmers, and others. Many studies have been conducted to measure reduction in crop growth during periods of soil moisture stress. The results of these experiments are varied due to the large number of factors, other than soil moisture status, that ultimately affect crop yield. Enough has been learned, however, about crop response to soil moisture stress to generally outline the yield response; but variations in the types and varieties of crops, yearly climate, soils, fertility levels, and cultural practices preclude precise definition. Additionally, difficulties in the exact measurement of soil moisture and climatic conditions make the mathematical specification of crop growth response difficult. Thus, a number of ways have been developed to specify crop response. All of the models discussed will have some divergence from the actual response of crops under most circumstances.

Part I reviews a number of models that have been developed to help predict crop response to soil moisture stress and to help plan efficient irrigation water allocation over the season and among crops.

Even with these rather elaborate models designed to determine optimum irrigation patterns, none of them addresses the problem of predicting precisely crop response to soil moisture stress by use of mathematical models. In order to determine what could be done in this area, a detailed analysis was undertaken in the Economics Department at Colorado State University to develop and test various mathematical models for suitability to predict yield response at various soil moisture levels during the growing season for specific crops. Detailed data from irrigation experiments designed to measure soil moisture status and its effect on yield throughout the season were used to test the models.

Two approaches using the agronomic experimental data were tried. Part II reports in detail on efforts to specify yield response to soil moisture stress.

One approach was used by Dr. Habte Neghassi in an effort to predict soil moisture levels by use of models utilizing evapotranspiration data and soil water-holding capacity. Soil moisture status is used to estimate resulting crop yield.

The second approach was developed by Dr. Dan Yaron using several mathematical models to estimate crop yield reductions when soil moisture falls below a predetermined level creating what is termed a "critical day." A critical day is one in which the crop suffers from moisture stress. Various mathematical functions were tested to determine if yield reductions could be predicted with some degree of confidence.

PART I. REVIEW OF IRRIGATION AND CROP RESPONSE MODELS

Various models of irrigation systems have been proposed with varying purposes in mind. Two basic types of models have been developed: irrigation water scheduling models and crop planning models. Scheduling models attempt to aid the farmer during the season, determining optimal timings and quantities of irrigation. Scheduling models keep track of some state variables related to plant growth and variables measuring water need and availability. These models are generally, but not necessarily, daily models.

Planning models are designed to aid farmers in choosing the best acreages of crops to be grown. The planning model must take into account resources known with certainty at the beginning of the season; these models must also deal in some way with such variables as precipitation, weather conditions including solar radiation, and stream flows which are known only probabilistically. Some form of scheduling model may be implicit in the planning model.

Simulation Models[1/]

<u>Jensen and Heerman</u>

Jensen and Heerman (1970) described an irrigation scheduling program that has been used by the United States Department of Agriculture and the Bureau of Reclamation, United States Department of the Interior, in advising farmers when to irrigate. The combination equation of Penman's evapotranspiration formula forms the basis of the program. Evapotranspiration, ET, is calculated on a daily basis from measured data and available soil moisture is updated by the program throughout the season.

1/ The basic summarization of the various models were done by Herbert Blank. A more detailed discussion of these and other models can be found in his Ph.D. dissertation, "Optimal Irrigation Decisions With Limited Water," Colorado State University, Oct. 1975.

At any time during the season, the next irrigation can be predicted using the formula:

$$N = \frac{D_o - D}{\bar{E}_t} \qquad (2.1)$$

in which

D = current estimated total depletion of soil moisture (in.)

D_o = maximum allowable depletion for the present stage of growth (in.)

$\bar{E}_t$ = mean daily ET rate for the 3 previous days and 3 forecast days (in./day)

N = estimated number of days to next irrigation.

In another paper (Heerman and Jensen, 1970), the $\bar{E}_t$ value used was obtained from a graph showing $\bar{E}_t$ as a function of time, normally distributed about the peak ET day. From experiments at Akron, Colorado, better results were obtained by this method than with the previous method, which required a subjective forecast of $\bar{E}_t$.

The next refinement in estimating the timing of the next needed irrigation was to add a term to N due to expected precipitation. The authors concluded that in a relatively dry area such as eastern Colorado, with relatively low precipitation, irrigation dates are not significantly affected by this refinement.

Kincaid and Heerman

Kincaid and Heerman (1974) describe a scheduling program for a programmable calculator. Again, the basis for the program is the Penman combination equation and associated crop coefficients and stress factors. As in the two previous papers, the authors assume the lowest soil moisture depletion level acceptable is 50 percent of the total available moisture within the root zone. At an irrigation, the soil profile is returned to field capacity. The method of forecasting the date of the needed irrigation uses a normally distributed $\bar{E}_t$ function.

The scheduling programs described have a specific purpose: recommending the timing of the next irrigation based on maintaining the crop within previously determined soil moisture conditions. The assumption, basically, is that water is available as needed and that no crop yield reduction is incurred when moisture depletion is not greater than 50 percent of available moisture.

Hanks

Hanks (1974) tested a production function for predicting grain yield from corn and sorghum. The author did not, however, attempt to apply this model in a planning or scheduling sense. The model is limited by data in that it requires daily values of potential evapotranspiration and potential soil evaporation under the crop canopy.

In a later paper, Hill, Hanks, et al. (1974) described a program which predicts corn yield using the production function tested by Hanks. The program was used to predict the effect of supplemental irrigation on an otherwise rainfed site. The conclusion was that a supplemental irrigation system could be economically justified. The program as described in the paper was used as a simulation of an irrigation system, answering a question "what if" irrigation were available.

Yaron

Yaron, et al. (1973) developed a soil moisture simulation model using experimental data from wheat. The authors fitted parameters to a Cobb-Douglas type function, an exponential function, and a Mitscherlich function. The Mitscherlich function was adopted having the following independent variables:

1. Number of days during growth season with soil moisture above about 45 percent of available soil moisture.
2. A variable which measured the quality of the germination period, and
3. A year variable (4 years of data were used in the regression).

Upon obtaining a suitable yield prediction equation, 16 years of rainfall data were used to simulate the effect on yield of two approaches to irrigation scheduling. These were:

1. Irrigation on the basis of a predetermined time schedule, the quantities of water applied being equal to the moisture depletion in the root zone at the time of irrigation, and
2. Irrigating at the date on which the soil moisture is depleted to a predetermined critical level (Yaron, et al., 1973).

Taking into account water costs, the conclusion is that the second policy is slightly better than the first according to three objectives: maximizing expected net return, minimizing variance, and maximizing income during years of low rainfall.

It should be noted that this is still a simulation approach; irrigation times and amounts were chosen according to two arbitrarily chosen rules and tested to determine net return.

<u>Stewart, Hagan and Pruitt</u>

Stewart, Hagan and Pruitt (1974b) describe 18 methods of corn production with limited water supply. These methods are derived from data from field trials at Davis, California. Four irrigation times were specified during the season and irrigations were applied in one-inch increments up to field capacity. The irrigations were scheduled to occur when 70 percent of the water applied previously had been removed from the root zone. A preirrigation to field capacity was made prior to planting. Yields were measured and profits due to water application were calculated, including water and labor costs of irrigating.

The authors recommended that if a fixed quantity of irrigation water per acre is known at the start of the season, the water should be applied according to the tables (see tables 4 and 5, Stewart et al., 1974b) derived by the authors. This model is thus deterministic and examines a single crop and an objective of maximizing return. The model could be adequate for the climatic conditions in the Central Valley of California, but is probably not readily transferable to other sites without repeating the full range of field trials.

Crop Optimization Models

The models discussed thus far have dealt with three aspects of the irrigation problem. The first studies were concerned with scheduling and, in particular, predicting date of next irrigation to obtain maximal yield. The second group was concerned with deriving production functions and then proceeding to simulate crop yields under varying conditions, while Stewart and Hagan's main contribution was in generating basic data relating water inputs to yields.

Hall and others have worked from the opposite end of the problem, starting with the optimization formulation and solution techniques, without concentrating on basic data.

Hall

Hall and Buras (1961) presented a problem of the otpimal crop acreage for a known limited water supply. They dealt with a single crop, for which return as a function of seasonal water input was known. The authors formulated a dynamic program to solve the problem and also developed a graphical solution technique. This model is limited in that it dealt with a single crop, was deterministic, and dealt only with the seasonal water input. The model did consider the problem of limited water supply, concluding that, at least in the concave region of the production function (Stage 2) the policy should be to irrigate the selected acreage uniformly. The selected acreage, apparently, depends on the shape of the particular production function.

Hall and Butcher (1968) introduced additional complexity by considering the effect of time of water application on yields. Again the model dealt with a single crop and again the top-down approach of assuming a production function was used. The form of the return function was

$$Z = P \prod_{i=1}^{n} a_i(d_i) \cdot Y_{max} - \sum_{i=1}^{n} c_i \cdot x_i \qquad (2.2)$$

in which

Z = return

P = price per unit of yield ($/lb.)

Y_{max} = maximum yield (lbs.)

d_i = soil moisture deficit from field capacity at time i (in.)

$a_i(d_i)$ = dimensionless yield reduction coefficient for time period i

x_i = quantity of water applied during period i (acre-inch)

c_i = cost of water application during period i ($/acre-inch)

After suggestions by Aron (1969) the model was presented in final form by Hall and Dracup (1970) as a dynamic program having three state variables which are

q = amount of water in storage (acre inch)

w = soil moisture level (in.)

and A = "state of the crop at any time as a result of the possible deficiencies before the time period" (Hall, 1969) (dimensionless).

The model may be classified as a single crop, deterministic, scheduling model. The model assumes a fixed supply of irrigation water to be applied to a known crop acreage. The results of the program are the optimal timings and amounts of irrigation water, determined on the basis of knowledge known at the beginning of the season. Precipitation and other random variables are apparently assumed to take on their mean values. The model is theoretical in that it is not based on actual data and is not applied to an actual site. In addition to the assumption regarding the multiplicative nature of the production function, the model assumes that daily evapotranspiration is a function only of the soil moisture level for that day, not of solar radiation, etc., though a more complicated relation could be adopted. Hall and Dracup (1970) discuss the problems of computation with a three-state variable dynamic program and suggest methods for speeding the program by restricting values of the state variables.

Minhas

Another single crop model was presented by Minhas, et al., (1974). They developed an evapotranspiration ET prediction model for wheat as a function of available soil moisture only. The function was of the form

$$f(x) = (1 - e^{-rx})/(1 - 2e^{-r\bar{x}} + e^{-rx}) \tag{2.3}$$

in which

r = parameter fitted from data (1/in.)

x = available soil moisture (ASM) in root zone (in.)

$\bar{x}$ = ASM at field capacity FC (in.)

f(x)= ratio of actual to potential ET for a plant when green cover is fully established.

Actual ET is the product of f(x); potential ET; and a crop weighting function, increasing from planting to full cover, constant until start of senescence, then decreasing to harvest. Parameters were fitted from wheat data from Delhi, India, and tested against results from alfalfa data of Mustonen and McGuinness (1968).

With an adequate ET prediction function, the authors used regression to fit parameters to the multiplicative function

$$Y = a\left[1 - (1 - x_1)^2\right]^{b_1}\left[1 - (1 - x_2)^2\right]^{b_2} \cdots \left[1 - (1 - x_n)^2\right]^{b_n} \qquad (2.4)$$

in which

Y = yield

x_j = relative (i.e., fraction of maximum) ET in period j

a, b_1, b_2, . . . b_n are positive parameters fitted from data. The data used were from 21 wheat experiments over 3 years. Dummy variables were introduced "to capture the effects of the differences in experimental designs, varieties used, amounts of fertilizers used, and the climatic factors (nonmoisture) between different years," (Minhas, et al., 1974). The resulting regressions generally had high values of R^2, but the parameters of interest tended to be nonsignificant.

The authors adopted a production function consisting of two time periods and formulated an optimization problem of maximizing yields subject to meeting a seasonal water availability constraint. The problem was solved via marginal analysis, equating marginal products of water in the two time periods.

Dudley

Dudley, et al., (1971a) formulated a two-state variable dynamic program to determine optimal timing of irrigation for corn with a limited seasonal water supply. The state variables were available soil moisture, average soil moisture, and quantity of water in storage. They assumed an additive growth function with varying dollar values for growth in each time period. A "growth-no-growth" assumption was made, employing a concept similar to the stress-day concept of Flynn and Musgrave (1967). If ASM is high in relation to potential ET, ET occurs at a maximum rate and a growth day occurs, contributing to the dollar value of the crop. If ASM is low, ET occurs at a rate E_m, "the maximum rate at which water moves into the plant from the soil mass," (Dudley, et al., 1971a) and a no-growth day is recorded, contributing nothing to the value of the crop.

A stochastic dynamic programming model was formulated to make use of 20 years of evaporation and precipitation data. The objective was to maximize expected return as a function of terminal soil moisture TSM, that is the ASM percentage at which an irrigation is to occur. Transition probability matrices of beginning soil moisture are generated for each TSM policy in each time period and for each level of water supply. Similar matrices are generated for beginning water supply and return.

The results of the stochastic dynamic program are employed in a second model described by Dudley, et al. (1971b). While the first model looked at optimal timing for a given acreage, the second looks at the optimal area to be planted to a single crop, adding an additional stochastic variable of reservoir inflow.

The problem solution technique is basically a simulation approach; an acreage is selected and expected return is calculated based on the 20 years of data and the optimal terminal soil moisture policies developed from the previous model. The process continues by varying the acreage and calculating return until an optimal return is achieved assuming return as a function of acreage is a unimodal function.

Anderson and Maass

The irrigation system developed by Anderson and Maass (1971, revised 1974) represents the next level of sophistication. This model simulates an irrigation system, including stream diversions and reservoir storage, water distribution rules used to operate the canals, individual farms of varying size, farm water supply and cropping patterns. Crop response to soil moisture conditions are simulated by specifying typical irrigation requirements by periods throughout the growing season. Up to 26 irrigation periods can be specified. Yield reductions are indicated for any missed irrigations. These yield reductions are estimates based on research of agronomists and others of the effects of water shortages on crop yield at various times during the irrigation season. Crop watering sequences are generated by use of one of the formulas specifying typical evapotranspiration demands for particular areas, the type of crop, stage of growth, expected precipitation and soil type. These, together with irrigation efficiency, determine the sequence and amount of water needed throughout the irrigation season.

A variety of rules have been programmed into the model to illustrate the various ways that the water supply of an irrigation system is distributed to farmers. These determine when and how much water a farmer will receive to irrigate his crops.

The model can be run in various ways. The first utilizes water supply data for a single season and runs it through the irrigation season to examine the yield results from a given water supply and fixed crop patterns on the farms. Results are for a particular season. This analysis shows the effects on individual farms and crops of a particular water supply using a particular distribution rule. Various water supplies and distribution rules can be compared this way.

A second way the model can be run is to use what is called the Plan routine of the program. This option allows the program to select within specified limits the optimum crop pattern for each farm given the seasonal water supply, the array of crops, its portion of the system's water supply, and crop yield responses to various irrigation sequences. The Plan routine selects the acres of various crops that can be grown to give the maximum return with water availability throughout the season. This is done by incrementing the highest return crops up to acreage or water limitations before bringing the next crop into the crop pattern.

Another way the program can be run is to use the same data as above but to institute various distribution rules to determine if there is a better way to distribute available water among farms in the system. This type of analysis can aid in estimating the efficiency of distribution rules.

<u>Young and Bredehoeft</u>

Young and Bredehoeft (1972) presented a multiple-crop planning model to determine a policy for conjunctive use of groundwater and surface water. Anderson and Maass considered several alternative methods of production for each crop. Young and Bredehoeft used the same idea, considering different amounts and timings of irrigation as different production methods. The optimal irrigation amount as developed by Anderson and Maass is one method; other methods correspond to skipping certain irrigations. Each method is associated with a certain net benefit per acre.

The model was simplified over Anderson's in that only four irrigation periods were considered. Groundwater was considered as an additional source of supply. A linear program was formulated similar to that of de Lucia except with the added dimension of time.

The irrigation planning problem was solved as a sub-program in a large simulation program. The authors did not consider the stochastic aspect of the problem due to the speed needed in computation. The authors restricted themselves to a site specific model with a single objective of maximizing return and all-or-nothing irrigations.

Hall

Hall and others in a report by the R.M. Parsons Co. (Parsons, 1970) applied Hall's work to a study of Indian irrigation. Data were obtained for two crops, wheat and jowar, and graphs were drawn for the coefficients $a_i(d_i)$ in the multiplicative yield function. For these two crops a dynamic program was developed to determine optimal timings and amounts of irrigation. Fertilizer was also considered, under the assumption that for a given water application, yields were related to relative quantity of fertilizer applied or

$$Y = a_N(N) a_1(d_1) a_2) \ . \ . \ . \ a_n(d_n) Y_{max}$$

in which $a_N(N)$ is given for maize by a graph. The program differed from that of Hall and Dracup (1970) in that the objective is to maximize yields and returns. Three state variables were considered: quantity of water in storage, soil moisture in the root zone, and available capital. The program allocates capital over the season between water and fertilizer. The results are optimal irrigation and fertilizer applications for a given level of available capital.

Various methods of production for the two crops are obtained from the dynamic program and these are used as input to a district-wide linear program that considers, deterministically, optimal crop acreages. The objective is to maximize the net value of the output. The constraints considered by this program are water availability in various time periods, land use constraints, fertilizer availability, manpower availability, and a constraint that limits the acreage of nonfood crops.

Discussion of Crop Optimization Models

Problem Statements

Young and Bredehoeft (1972), Anderson and Maass (1971), Hall (Parsons, 1970), and de Lucia (1969) all consider basically the same problem: maximizing yearly yields or return from a fixed irrigated acreage, considering a given number of feasible crops. Smith (1970) is concerned with maximizing the net present worth of a planned expansion of a presently irrigated area, considering capital investments of the project and capacity dependent operation and maintenance costs, in addition to costs of water.

All of the previously mentioned authors consider linear constraints such as land constraints, water use constraints, etc. Smith (1970) and Hall (Parsons, 1970) consider crops grown in time periods extending throughout the entire year, but none of the studies considers more than one year and possible crop rotation requirements.

Basic Data

The data used by the authors range from being based on extensive field trials to being based on rather questionable assumptions. Stewart and Hagan (1973b) conducted field trials, growing corn under many different irrigation regimes.

Yaron, et al., (1973) and Minhas, et al.,(1974) rely on data from a number of years to establish their respective production functions. A "year" term is often included in the regressions. When the year term accounts for much of the variation in observed yields, the model obviously has not been well constructed. A model of plant growth which includes soil moisture and climatic terms should not require a year term. Another alternative is to use data collected in a single year, thus eliminating complicating effects of climatic variability.

Several of the authors devote little time to discussing the data on which their studies are based. Consumptive use figures for fully watered crops are available for many crops in many locations. These data are adequate for a study such as de Lucia's (1969). In other studies, including Hall's and Anderson and Maass', it appears that data for yields under conditions of less than optimum water supply have been based, in some cases, on judgment resulting from limited observations. This is not meant to be a criticism of the studies, only a reflection on the lack of data and the lack of theory to predict crop yields, These models have turned to substitutes for actual crop response data because of the extreme complexity and interaction of crops, growth stage, soil characteristics, atmospheric conditions and variation in water availability.

Growth Models and Production Functions

Similar to the diversity in making use of basic data, diversity was noted in the growth models and production functions adopted by the various authors.

Stewart and Hagan (1973b) proposed a growth model linearly relating yields to seasonal ET. Jensen and Heerman (1970) and Hanks (1974) have complicated models for predicting ET. Hanks related ET in various time periods to yields with a multiplicative function. Hall used a multiplicative production function with terms functions of soil moisture during the time periods. Updating soil moisture in Hall's model requires predicting ET. Hall's ET (Hall and Butcher, 1970) is only a function of available soil moisture, ASM.

In the model of Minhas, et al., (1974), ET is a function of ASM, potential ET and a crop factor. Evapotranspiration is related to yields through a multiplicative production function. Dudley, et al., (1971a) predict actual ET from free water evaporation, a crop factor, and a soil factor. Yields are predicted based on the growth-no-growth concept which is based on daily ET values.

All of the previously mentioned authors rely on an ET estimation model. Some authors relate ET to yields while others, such as Hall, require estimates of ET in order to update soil moisture, which in turn is related to the yield coefficients in each time period. In any case, an ET estimation model is needed.

Additive versus Multiplicative Functions

Multiplicative production functions have been employed by Hanks, Hall, and Minhas. Jensen (1968) proposed using the multiplicative relation for some crops, but the irrigation scheduling programs of Jensen assume only one method of production. Anderson and Maass and Young and Bredehoeft do not employ continuous production functions.

Smith, in his simulation model, assumes a "linear relationship between crop yield and the water applied during any decision period" (Smith, 1970). An additive function, based on theory by Moore (1961) does not appear to be justified for all crops (Hall and Dracup, 1970, p. 134; and Jensen, 1968). Dudley's growth-no-growth concept is an additive relation with each growth day contributing a dollar value to the crop.

The multiplicative relation implies, for example, that if growth is only 70 percent of potential for a particular growth stage, then the maximum yield attainable by the crop is 70 percent of potential. According to the additive theory 70 percent of potential growth in a particular time period will only result in potential yields being reduced by 30 percent of that particular time period's potential contribution (see Figure 1).

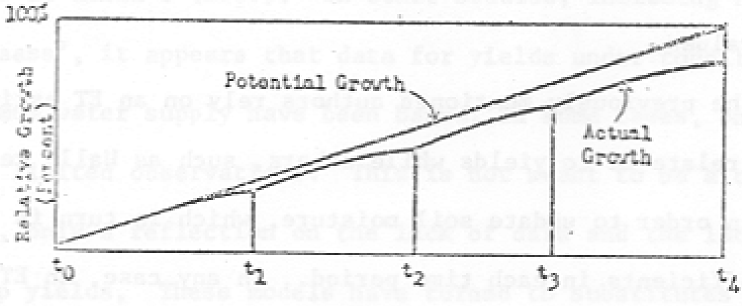

Figure 1

ACTUAL AND POTENTIAL GROWTH BY IRRIGATION CYCLES

(From Moore, 1961)

Again, the point is made that no adequate theory has been developed and that currently available data are not sufficient to conclusively adopt any of the production functions described. As new data become available there is a need to test them with both approaches.

PART II. MODELS DESIGNED TO SPECIFY SOIL MOISTURE STATUS AND YIELD RESPONSE

To explore the problems and difficulties of applying experimental water use data to models that are designed to predict or explain yield response to soil moisture stress on crops throughout the growing season, two different approaches were used to test various predictive models on corn and sorghum experiments. The first attempt was by Dr. Habte Neghassi testing several models against observed soil moisture use on corn at the Colorado State University agronomy farm in Fort Collins.

The second attempt was more elaborate and was made by Dr. Dan Yaron and colleagues to predict soil moisture status and yield response of corn from Fort Collins data. These models use data obtained from experiments designed to determine crop response to various levels of soil moisture availability throughout the growing season.

These exercises are presented to show the difficulties encountered when attempting to develop predictive models. The Yaron method does give guidelines for predicting yield reductions in corn.

CORN WATER USE AND YIELD MODELS (Dr. H. Neghassi and Dr. R. Young)

Objective

The broad objectives of this analysis were to simulate water use and response models for various crops using historical data. However, due to limitations and unsuitability for combining the data, only corn grown at Fort Collins, Colorado was studied.

Crop, Soils, and Climatic Data

Corn (Zea Mays L.) was grown at the Colorado State University Agronomy Farm. The study was conducted by Twyford (Twyford, 1973) under the supervision of Dr. R.E. Danielson in 1972. The crop, planted May 12, was grown under varying soil moisture regimes. There were 11 treatments, representing three irrigation quotas, involving three schedules each, one irrigation quota involving one schedule, and one control. All irrigation treatments received water during the critical silking period. All irrigation applications were 5 cm (2 inches) by basin irrigation. The schedules refer to length (days) of irrigation delay during silking.

The soil was uniform deep Nunn Clay loam. There were three plant densities of low 54,000, medium 69,000, and high 85,000 plants per hectare. Uniform 47 kg/ha Phosphorous and 107 kg/ha Nitrogen were applied. The ultimate root depth was 195 cm with total water holding capacity of 26.6 cm.

Soil moisture was measured using a neutron probe at intervals during the growing season. Only the medium population density plots were sampled. No measurement of ground water level in the root zone was made but probably did not exist.

Daily climatic records of maximum and minimum air temperatures, precipitation, and minimum relative humidity were recorded at the experimental site. No records of wind speed, saturation vapor pressure, and solar radiation (or percent sunshine) were made. Adaption of solar radiation measurements at the Horticulture Farm, Colorado State University, which is located about 7 miles NNE of the Agronomy Farm, made the climatic data suitable for estimating potential evaportranspiration using the Jensen-Haise method. The solar method malfunctioned many times during the growing season. Measurements indicate obvious overestimation even under clear skies.

Data gathered by Dr. R.E. Danielson in 1968 were also analyzed. The experimental objective and design were not the same as the 1972 experiment.

Estimation of Evapotranspiration

Potential evapotranspiration

The climatic input was incomplete to estimate potential evapotranspiration, ET_p, by the combination, or Penman, method (Penman, 1963), which would have been preferred. Thus, ET_p was estimated by the Jensen-Haise method (Jensen and Haise, 1963), which requires average daily temperature and solar radiation as input. The equation is given by

$$ET_p = (p.025T_a + 0.080)R_s \;.... \qquad (1)$$

where T_a is the average daily temperature in ^oC, R is the total short wave radiation in cal $cm^{-2} day^{-1}$ received from the sun and the sky, and ET_p is cm day^{-1}.

Actual evapotranspiration

Daily evapotranspiration for a given agricultural crop under actual conditions of soils and climate, ET, is related to daily potential evapotranspiration, ET_p, as follows:

$$ET = K_c ET_p \qquad (2)$$

where K_c is a dimensionless coefficient. It represents the combined relative effects of the resistance of water movement from the soil to the various evaporating surfaces and the resistance to the diffusion of water from the surface to the atmosphere, as well as the relative amount of radiant energy available as compared to the reference crop. The crop coefficient derived from conditions of water non-limiting is designated by K_{co}.

In the USDA irrigation scheduling computer program the crop coefficient is adjusted for soil water availability and soil surface wetness as follows:

$$K_c = K_a K_{co} + K_s \tag{3}$$

where K_a is the relative coefficient related to percent available soil water, AM, as follows:

$$K_a = \ln(AM + 1)/\ln 101 \tag{4}$$

K_s is the increase in the coefficient when the soil surface is wetted by irrigation or rain. It is approximated by:

$$K_s = (0.90 - K_c)m \tag{5}$$

in which m = 0.8, 0.5, or 0.3, respectively, for the first, second, or third day after irrigation or rain. In this particular case, K_s = 0.8, 0.7, or 0.6 when the rain or irrigation exceeded 1.5 cm for the first, second, and third days.

The mean crop coefficient where soil moisture was not limiting and normal irrigation stands are used, K_{co}, varies with type of crop. For corn, K_{co}, is given by

$$K_{co} = 0.23 - 0.4276P + 2.756P^2 - 1.583P^3 \tag{6}$$

where P is the fraction of days from planting to time of heading. After heading, K_{co} is given by

$$K_{co} = 0.915 + 1.195 - 4.688D^2 + 2.75D^3 \tag{7}$$

in which D is the number of days after heading divided by 100.

For this case, K_{co} was kept at 1.00 for the first 40 days after heading, or until $D \geq 0.40$.

Soil moisture depletion

The major dependent variable is soil moisture depletion and the major components are:

$$DSW = \sum_{i=1}^{n} (ET - R_e - I + W_d) \quad (8)$$

where DSW is soil moisture depletion (after a thorough irrigation D = 0), R_e is effective rainfall (excluding surface runoff), I is irrigation water applied, and W_d is drainage from the root zone. The terms to the right of the equal sign are daily totals, expressed in cm, in the present computer program.

The amount of water available in the root zone (holding capacity = 26.6 cm) at any time during the growing season is given by:

$$ASW = 26.6 - DSW \quad (9)$$

where ASW is available soil water.

Comparison of estimated and measured water use

Available soil water was selected as a criterion for comparing the estimated and measured water use. Microfilm plats of the measured available soil water (points) and estimated available soil water are presented in figures 2 and Appendix B, figures 1-10, one for each irrigation treatment. Soil water measurements were first made on June 22 (Julian day 173). This measurement is taken as the initial soil water level for the simulation, and thus is implicitly assumed correct. The estimate of soil water between planting (May 12, Julian day 132) was made by reading in an initial value for available soil water which would, after considering the various components of depletion, give close correspondence to the measurement of June 22.

The measured and estimated soil water availability compare well for the drier treatments, 0, 1A, 1B, and 1C (fig. 2 and Appendix B figures 1-3). Treatment 0 received no irrigation and 1A, 1B, and 1C received one 5 cm irrigation. Treatment 3C (fig. A6) also gave close agreement.

As shown in Appendix figures A4, A5, A6, A7, A8 and A9, treatments 3A-B, 4A-C, and 5 compare very poorly. The measured available soil water level is consistently lower than that estimated. Some possible causes for the discrepancies are:

1. Error in measurement (Neutron probe). Some of the measurements were obviously in error and reasonable adjustments were made in such cases.

2. High advective energy causing water losses much higher than a normal field would experience. The plots were separated by dry boundaries, which would increase advective loss.

3. Lateral and vertical movements of soil water from the root zone. These were not measured.

4. The solarimeter obviously malfunctioned occasionally during the season. It was overestimating solar radiation indicating higher values than would be expected on clear days at this

latitude. The solar radiation measurements were construed to be less than or equal to 850 cal cm^{-2} day^{-1}.

5. The crop coefficient tended to underestimate the daily estimates. The length of the period to heading of 85 days suggested by Jensen appeared to be too long. This was reduced to 45 days after consultation with Dr. Cuany of the Agronomy Department. It was reasonable to assume that the leaf area index reaches 3 after 45 days. Another adjustment was also introduced to account for advective losses for the first three days after irrigation or rain.

Another way of comparing the estimated and measured values is by use of the seasonal balances given in table 1. The 1972 data show that the measured water use by treatments 3A-C, 4A-C, and 5 was higher than the potential. This is unreasonable. The estimates of potential ET appear to be very low. The most probable cause may be that the solar radiation measurements were low. The measurements were obviously in error in 1968, since the treatments with the most irrigation result in lower seasonable measures than the dryer ones. The ground water level was high according to the incremental measurements in the root zone.

Below the third foot in the 100 cm. zone, the plots were at field capacity throughout the season. Thus, the 1968 data were not analyzed further. The 1972 data were used in developing yield models.

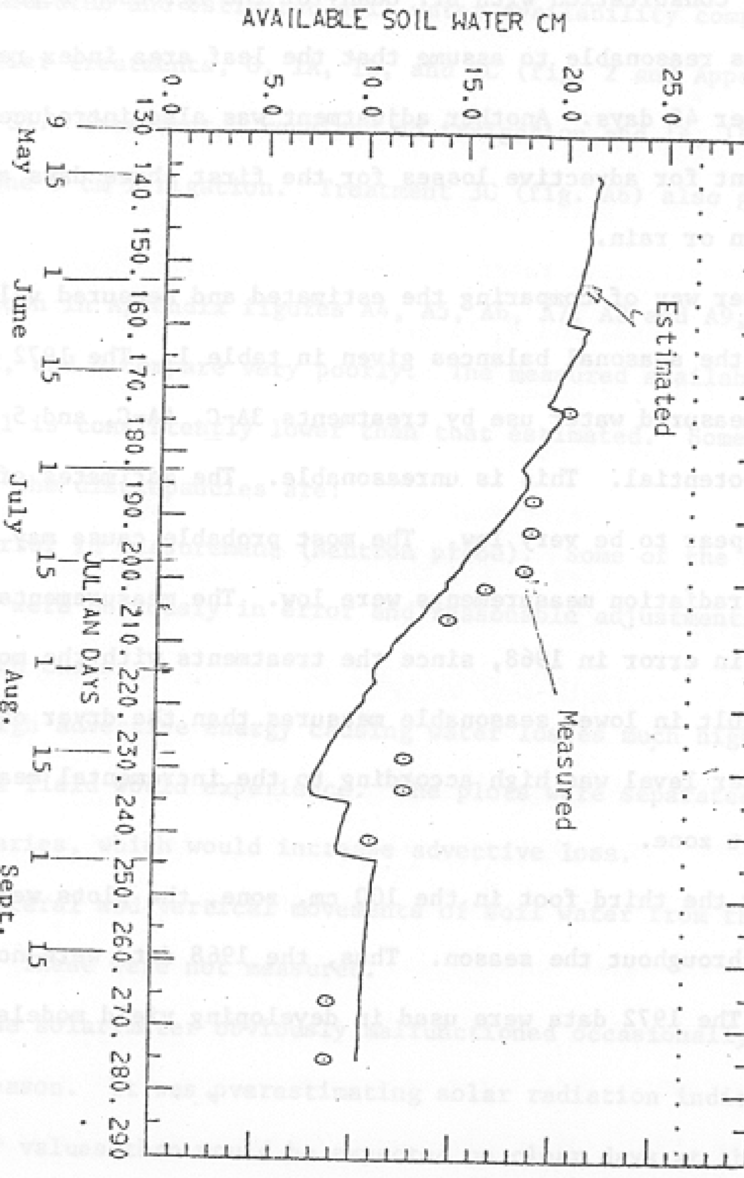

FIG. 2--TREATMENT-0--NO IRRIGATION

TABLE 1

SEASONAL ESTIMATES OF WATER USE, RAINFALL AND IRRIGATION FOR CORN IRRIGATION TREATMENTS, AGRONOMY RESEARCH FARM, 1972

Treatment	Water Estimated (cm)	Use Measured (cm)	Potential ET (cm)	Transpiration (cm)	Rain and Irrigation (cm)	No. of Irrigation*
			1972			
0	21.73	22.35	30.34	15.61	9.46	0
1A	22.90	24.76	30.34	15.93	14.46	1
1B	23.10	25.46	30.34	16.22	14.46	1
1C	22.12	23.96	30.34	15.78	14.46	1
3A	23.80	34.16	30.34	17.14	24.46	3
3B	23.85	32.96	30.34	16.96	24.46	3
3C	25.20	33.26	30.34	17.80	24.46	3
4A	24.32	37.08	30.34	17.44	29.45	4
4B	24.00	35.22	30.34	17.40	29.46	4
4C	24.15	30.94	30.34	17.72	29.46	4
5	24.66	30.74	30.34	17.85	34.46	5
			1968			
I	13.75	13.83	16.79	10.63	34.36	4
II	13.88	18.31	16.79	10.41	26.88	3
III	13.86	24.79	16.79	10.39	24.51	2
IV	13.37	20.80	16.79	9.95	17.54	1
V	12.46	27.68	16.79	9.23	11.92	0

*The amount was 5 cm per irrigation in 1972. It was variable in 1968.

Crop Response to Selected Yield Indicators

A. Models

Selected seasonal and intra-seasonal yield indicators were used to evaluate parameters for production functions for corn. The following functions were formulated:

$$Y = Y_{max}(B_1)^{X_1} B_2^{X_5} \quad (10)$$

$$Y = Y_{max} B_2^{X_2} B_2^{X_5} \quad (11)$$

and

$$Y = Y_{max}(1 - B, e^{-B_2 X_3}) B_3^{X_5} \quad (12)$$

In the above expressions, Y is grain yield in kg ha^{-1}, Y_{max} is the maximum yield, X_1 is the number of days soil moisture depletion was greater than 12 cm between June 22 and October 2. X_2 is the number of three-day periods that soil moisture depletion was above 12 cm, X_3 is the number of days soil moisture depletion was below 12 cm during the season, X_4 is the same as X_1 but applies to the silking period only. X_5 is the number of days irrigation has been delayed after silking began, and the B's are parameters. Note that Y_{max} is also a parameter.

Equations 10 - 11 are power (exponential) models. Equation 12 interacts the Mitscherlich (Heady and Dillon, 1961) and the power functions. Definitely both are non-linear. Note that the form of the power models require yield decreasing measures, whereas the form of Mitscherlich model requires a yield increasing measure.

Attempts were made to estimate the parameters in the above equations using STAT 31R (CSU) which is a computer algorithm for non-linear least squares estimation. The algorithm is based on the Marquardt's compromise (Marquardt, 1963) which represents a compromise between the steepest descent and linearization methods.

B. Convergence problem

The iterative process involved in deriving the parameter in Model 13 leads to non-convergence. Constraining the parameters B_1 and B, such that both live between 0.0 and 1.0 lead to estimates of Y_{max} and B_2 that were extremely large and non-realistic. The power functions converged at relatively few iterations, the maximum being 6, which is very efficient. The speed of convergence depends on the closeness of the initial guesses to the solution values.

C. Results

The data basis for fitting the models is presented in table 2. Estimates of the parameters in the power (exponential) models 10-11 are summarized in table 3.

The estimates of B_1 and B_2 fell between 0 and 1.0 as would be expected. An increase in the variables X_1, X_2, and X_5 reduces yield. The rate of reduction is:

$$\frac{\partial Y}{\partial x_i} = Y \ln B_j \qquad (13)$$

i = 1, 2, 5 and j = 1,2

The coefficients of determination apply to linear transformation of the models. This procedure was applied to get an idea of the explained variation in terms of standard linear analysis. The non-linear technique uses other criteria for convergence and R^2 is not given. However, it should be greater than the indicated value.

TABLE 2

CORN GRAIN YIELD AND RELATED YIELD INDICATORS FOR WATER PRODUCTION FUNCTIONS
AGRONOMY RESEARCH FARM, COLORADO STATE UNIVERSITY, FORT COLLINS, COLORADO
1972

Treatment	Grain Yield for Indicated Treatment Population Densities L (kg/ha)	M (kg/ha)	H (kg/ha)	Soil Water Related Yield Indicators*: Seasonal DSM** - 12cm 1 Day Period X_1	Seasonal DSM** - 12cm 3 Day Period X_2	DSM - 12cm 1 Day Period X_3	Silking DSM - 12cm 1 Day Period X_4	Irrigation Delay Days X_5
0	5140	4138	4030	-	-	-	-	-
1A	7823	7128		72	24	30	8	0
1B	7080	7142		61	14	41	5	5
1C	6873	6817		70	23	32	1	10
3A	9164	9450	8696	61	16	41	5	0
3B	8834	9037	9028	58	17	49	3	2
3C	8267	8906	8777	4	1	99	4	7
4A	10363	10130	9973	31	8	71	0	0
4B	9537	9972	9611	21	7	81	0	3
4C	8879	9553	10107	0	0	102	0	6
5	10119	9799	10035	0	0	102	0	2

* Obtained from corrected plot of soil water depletion over time.

** DSM = Depleted soil moisture.

TABLE 3

Non-linear Least Squares Estimate of Parameters in Power Function (Models 10 - 11) for Corn, Agronomy Research Farm, Colorado State University, Fort Collins, Colorado, 1972

	Model	Parameters Related to Indicated Variables[1/] Y_{max}	X_1	X_2	X_3	X_5	Standard Error of Estimate	R^2
	10	10959.2	0.996			0.973	551.1	0.76[2/]
S.E.[3/]		275.1	0.0005			0.004		
N.C.I.	Upper	1140.7	0.997			0.983		
	Lower	1051.1	0.995			0.963		
	11	10838.7		.989		.974	584.8	0.74
S.E.		280.5		0.002		.005		
N.C.I.	Upper	11309.4		0.992		.984		
	Lower	10368.1		0.985		.964		
	12		Did not converge					

1/ For definitions, see Table 2.

2/ R^2 was estimated by linearization. The non-linear R^2 must be greater than indicated.

3/ SE = Standard error of parameter estimates.

N.C.I. = Non-linear confidence limits.

Conclusion

These models are suitable for simulation of crop responses if accurate measurements of the soil water status can be made. The parameters given here need to be reevaluated in light of the fact that the soil water measurements employed here appeared to have errors.

EMPIRICAL ESTIMATES OF RESPONSE FUNCTIONS OF CORN TO SOIL MOISTURE STRESS

(Dr. Dan Yaron)

Introduction

This study uses empirical estimates of response functions of corn to soil moisture stress. These estimates are based on two corn irrigation experiments at the Agronomy Research Station, Colorado State University, Fort Collins, in the years 1972 and 1968.

In the expression of the soil moisture variables the concept of "stress days" or "critical days" was applied. A "critical day" was defined as one in which the soil moisture in the root zone was depleted below a certain level (45-55 percent of the available soil moisture, AMS). The number of "critical days" thus defined, or, alternatively the number of "noncritical" or "growth" days were used as explanatory variables in the response functions.

Two general formulations of the response function were applied: (a) the Mitscherlich function; and (b) the exponential function. The specific forms of these functions, which were applied, and their interpretation are discussed, along with the empirical estimates, in the following sections.

Response Function of Corn to Soil Moisture, Ft. Collins, 1972

The Experimental Data

Irrigation experiments which provided the data for the analysis were conducted in 1972 at the Colorado State University Agronomy Research Station, and were studied by Twyford (1973). Corn was planted on May 11 and harvested on October 26. 2/ There were 11 treatments in the experiment varying with time and number of irrigation applications. Each irrigation

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2/ Final harvest.

applied water to a depth of 2 inches (5 cm.). The sequence of irrigation treatments is shown in table 4. Yields from the zero quota plots, those that received no water, were excluded from the analysis due to the possible contribution of ground water to soil moisture in the experimental plots of this treatment.

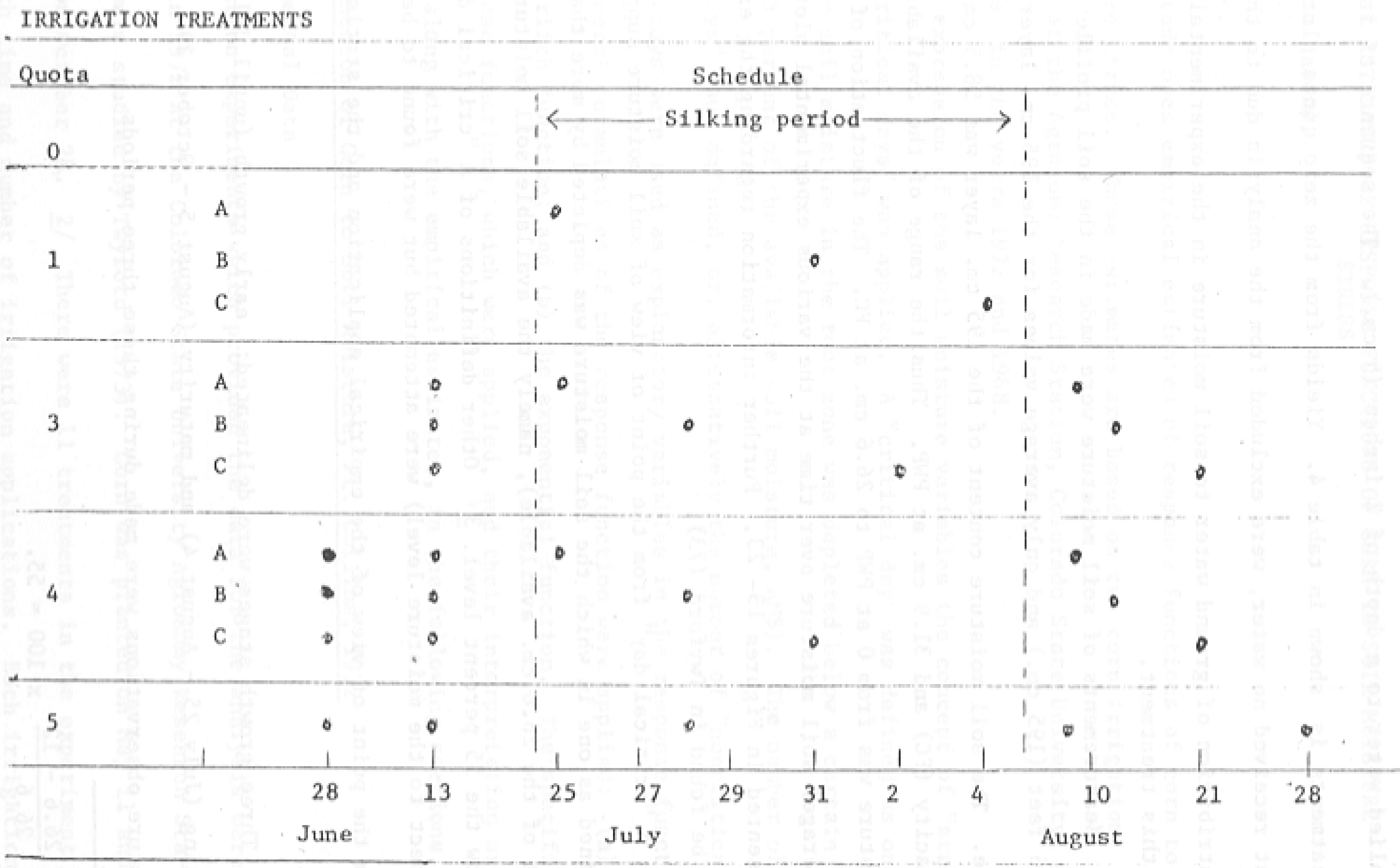
Measurements of soil moisture were made in the soil profile to a depth of 6 feet (195 cm.) and only average values for the 195 cm. layer are available. The soil moisture content of the 195 cm. layer was 58.5 cm. at field capacity (FC) and 31.9 cm. at PWP. Thus the range of the available soil moisture was from 0 at PWP to 26.6 cm. at FC. The fluctuation of the (average) soil moisture over time at the various experimental plots are presented in Figures 13- 23. Further information regarding the experiment can be found in Twyford (73).

A "critical day" from the point of view of soil moisture supply was defined as one in which the soil moisture was depleted by more than 12 cm. (out of the 26.6 cm. available), namely the available soil moisture dropped below the 55 percent level. 3/ Other definitions of a "critical day" (with respect to the moisture level) were attempted but were found to be inferior from the point of view of the empirical application and the statistical fit.

Three growth stages were delineated: early growth (until July 24), silking (July 25 - August 4) and maturity (August 5 - October 2). Soil moisture observations were made during these three periods.

3/ $\frac{26.6 - 12}{26.6} \times 100 = 55.$

Table 4--Irrigation treatments, corn irrigation experiment, Fort Collins, Colorado, 1972



The Mitscherlich (modified) function

One specification of the response function attempted (a combination of the Mitscherlich and the exponential forms) was the following:

(1) $y = A(1 - Be^{-kx_1^*})C^{x_2}$

where

y = grain yield of corn /kg/ha/,

x_1^* = relative number of "non-critical" growth days in the non-reproductive period (early growth and maturity stages) with soil moisture above 55 percent of ASM, expressed as the percentage of the total number of days in this period.

x_2 = the number of critical days during the silking stage.

A, B, k and C = the parameters estimated.

The estimates of (1) were obtained using a steepest descent (computerized) search method intended to minimize, or rather to obtain low values of $\Sigma(y_i - \hat{y}_i)^2$. The "best" estimates in terms of $\Sigma(y_i - \hat{y}_i)^2$ and general soundness of the results are presented in table 5.

The estimated values of $\hat{y}$ versus the actual yields using estimate No. 1 are presented in table 6, and those using estimate No. 2 are presented in the Appendix in table A1.

The interpretation of Estimate 1 suggests that:

(a) The asymptotical yield ($\hat{A}$) is 10,000 kg/ha approximately (161 bu./acre).

(b) Each critical day in the silking period reduces the yield by 2 percent approximately ($\hat{C}$ = .98).

(c) Under conditions of $x_1^* = 0$ the maximal yield will be reduced by 90 percent. Note that the value of $\hat{B}$ from which this result is derived was imposed. In Estimate 2 with $\hat{B} = 1$, for $x_1^* = 0$, the yield is reduced by 100 percent.

Table 5--Empirical estimates of the Mitscherlich type response function for corn, Ft. Collins, 1972

Estimate number	Estimated parameters $\hat{A}$	$\hat{B}$	$\hat{k}$	$\hat{C}$	$\bar{R}^2$ 2/
1	9,957	1/ 0.90	5.529786	0.98163	0.75
2	9,946	1.0	5.820091	0.98184	0.75

1/ With the value of $\hat{B}$ being imposed to equal 0.90.
2/ Computed as $\bar{R} = \frac{\Sigma(y_1 - \hat{y}_1)^2}{\Sigma(y_1 - \bar{\bar{y}}_{yi})^2}$ with the conventional notation.

* * *

Table 6--Actual and estimated values of corn yield using estimate No. 1, Ft. Collins, 1972 irrigation experiment

Treatment	Variables' values 1/ x_1	x_2	y_i	$\hat{y}_1$	$(y_i - \hat{y}_i)$	Relative deviation, percent 2/
1A	.27	6	7,128	7,107	21	0
3A	.42	7	9,450	7,974	1,476	16
4A	.65	0	10,130	9,711	419	4
1B	.36	5	7,142	7,960	-818	-11
3B	.47	2	9,037	8,953	84	1
4B	.84	0	9,972	9,871	101	1
1C	.38	9	6,817	7,499	-682	-10
3C	1.00	5	8,906	9,043	-137	- 2
4C	1.00	0	9,553	9,922	-369	- 4
5	1.00	0	9,799	9,922	-122	- 1

1/ See text for the definition of the variables.
2/ Computed as $\frac{(y_i - \hat{y}_i)}{y_i} \times 100.$

(d) In order to obtain the marginal productivity of x_1 we define

(2) $\hat{A}* = \hat{A}\,\hat{C}^{x2}$

and

(1) $\hat{Y} = \hat{A}* \,(1 - \hat{B}e^{-kx_1^*})$

and take the partial derivative of (1)' with respect to x*, to obtain:

(3) $\frac{\partial y}{\partial x_1^*} = \hat{k}\,(\hat{A}* - \hat{y})$.

Values of the marginal productivity of x_1^* based on Estimate No. 1 for selected situations are presented in table 7, and those based on Estimate No. 2 are given in table A2 in the Appendix. Note that the range of observations was $.27 \leq x_1^* \leq 1.00$, $0 \leq x_2 \leq 9$, and $6,817 \leq y \leq 10,130$.

Examination of table 4 suggests that the marginal contribution of x_1^* is in the range of between 0.2 to 9 percent of the maximal yield or between 2 to 90 kg/ha. The results expressed on the basis of the actual rather than relative number of non-critical days are much alike since there were 92 days in the non-silking period. The marginal contribution of x_1^* at the mean of x_1^* is 1.4, 1.3 and 1.2 percent of the maximal yield for $x_2 = 0$, $x_2 = 5$ and $x_2 = 10$, respectively.

The exponential function

Another specification of the response function was the following:

(4) $y = A\,b_1^{x1}\,b_2^{x2}$

with

x_1 = the number of critical days in the non-reproductive period (soil moisture below 55 percent of AMS);

y, x_2 = as previously defined;

A, B_1, b_2 = parameters ($0 < b_1, b_2 \leq 1$).

Table 7--Marginal product of x_1^* for selected combinations of x_1^* and x_2^*, based on Estimate 1

Variables' values 1/			:	:	
x_1^*	x_2	$\hat{y}$	$MP_{x_1^*}$ 2/		Relative marginal product 3/
.30	0	8,251	94		9.4
.30	55	7,521	86		8.6
.30	10	6,855	78		7.8
.50	0	9,393	31		3.1
.50	5	8,561	28		2.8
.50	10	7,803	26		2.6
.75	0	9,816	7.8		0.78
.75	5	8,946	7.1		0.71
.75	10	8.154	6.5		0.65
1.00	0	9,922	1.96		0.196
1.00	5	9,043	11.79		0.179
1.00	10	8,242	1.63		0.163

1/ See text for the definition of the variables.

2/ Computed according to (3) in text as $\frac{\partial \hat{y}}{\partial x_1^*}$ =

3/ Computed as $(MP_{x_1^*}/1\hat{A}) \times 100$.

The estimate obtained by regression techniques was:

$$(5)\ \hat{y} = \underset{**}{10,070} \cdot (0.9983)^{x_1} \underset{**}{(0.975)}^{x_2} \quad R^2 = 0.65$$

with ** denoting significance of the estimates parameter at 1 percent, and § denoting nonsignificance at an acceptable probability level.

The interpretation of the above estimate is as follows:

(a) The asymptotic yield is 10,000 kg/ha approximately ($\hat{A} = 10,070$);

(b) Each "critical day" in the silking stage reduces the yield by 2.5 percent ($b_2 = .975$);

(c) Each "critical day" in the nonreproductive period reduces the yield by 0.27 percent approximately ($\hat{b}_1 = .9983$).

Note that the latter result was derived from the estimated value of b which was found to be "non-significant", i.e., subject to a considerable error of estimate.

In another attempt to evaluate the effect of x_1 Figure 11 was drawn, showing the relationship between y and x_1. The figure suggests that for each of the three groups of treatments A, B, C (early, medium and late irrigation during the silking stage) there exists a response function (free-hand drawn on the graph), with only one observation in the C group (3C) diverging from the otherwise quite regular pattern. In view of this observation two separate functions of the form

$$(6)\ y = A\, b_1^{x_1}$$

were estimated for the A and B groups. Note that there were only three observations in each group and the formal statistical significance of the estimates is rather dubious. Nevertheless, the estimates obtainec provide a notion of the effect of soil moisture on the yield in the non-productive period.

The estimates were:

$$(7) \quad \text{Group A:} \quad \hat{y} = 14{,}250\ (0.9904)^{x_1}$$

$$\text{Group B:} \quad \hat{y} = 11{,}210\ (0.9936)^{x_1}$$

The above estimates suggest that each critical day in the non-reproductive period reduces the yield by 0.7 - 1.0 percent.

Estimates of (6) with imposition of $\hat{A} = 10{,}000$ were

$$(8) \quad \text{Group A:} \quad \hat{y} = 10{,}000\ (0.9960)^{x_1}$$

$$\text{Group B:} \quad \hat{y} = 10{,}000\ (0.9968)^{x_1}$$

indicating that each critical day in the non-reproductive period reduced the yield by 0.3 - 0.4 percent.

Conclusions

It seems that the following conclusions can be derived from the analysis of the 1972 Ft. Collins corn irrigation experiment:

(a) The concept of "critical days" (or non-critical ones) provides a valid basis for the definition of explanatory variables in the specification of response of corn to soil moisture.

(b) Any critical day in the silking period, here defined as a day with soil moisture lower than 55 percent of ASM in the root zone reduces the yield by 2 - 2.5 percent.

(c) Any critical day in the non-reproductive stages of growth reduces the yield by a fraction which has not been uniquely estimated on the basis of the experimental data available. The estimates derived using the Mitscherlich function indicate a reduction factor varying between 9 percent and 0.2 percent of the maximal yield per each critical day. For the mean value of x^* (= .645) the reduction per day is 1.2 - 1.4 percent. The estimates derived using the "nonrestricted" exponential function (7) indicate a reduction of 0.7 - 1 percent, and with the imposition of A = 10,000 as in (8) a reduction of 0.3 - 0.4 percent. Additional information is needed to improve these estimates. 4/

Finally, it should be noted that the definition of a "critical day" applied in the above analysis is somewhat arbitrary. No significant difference between the 55 percent of AMS as the critical level versus, say 50 percent of AMS, could be claimed. Note that the number of days with soil moisture below .55 AMS is highly correlated with that below .50 AMS. Further information from plant physiologists and soil scientists is needed in order to define the critical level more precisely.

Response Function of Corn to Soil Moisture, Ft. Collins, 1968

The Experimental Data

The experiment analyzed in the following was conducted in 1968 at the Agronomy Research Station, Colorado State University on Nunn clay loam soil. The field was planted with Kitely K4-17 (105 day season) hybrid seed on May 9. The experiment involved two factors, namely soil moisture (irrigation) and nitrogen fertilizer. The experimental design and treatments, the irrigation schedule, the rainfall records, and the grain yield are shown in tables 8 through 11 respectively.

Measurements of soil moisture tension were taken throughout the major part of the season (July 3- August 28) for the three upper soil layers of one foot depth each. Soil moisture tension of ½ bar, equivalent to 26.1 percent of soil moisture (on gravimetric basis) was considered as field

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4/ Note that (7) and (8) yield a compounded reduction rate. For the mean number of 64 critical days compounded reduction rate of 0.5 percent is equivalent to a non-compounded rate of 1.1 percent ($0.995^{64} = 0.011 \times 64$).

Table 8--Experimental design and treatments, corn, Fort Collins, Colorado, 1968[1/]

Nitrogen fertilizer treatments (lbs./acre)	Irrigation treatments				
	I	II	III	IV	V
200	2		2		2
150		1		1	
100	2		2		2
50		1		1	
0	2		2		2
	Maximum soil water tension (in Bars)[2/]				
	0.7	1.0	3.0	6.0	9.0

1/ Figures represent the number of replications in each of the blocks.
2/ Maximum soil water tension in Bars allowed at 12-inch soil depth.

* * *

Table 9--Irrigation schedule, corn, Fort Collins, Colorado, 1968

Date	Irrigation treatment				
	I	II	III	IV	V
	Inches of water/irriation				
July 5	2.57				
July 11		2.02			
July 15			2.94		
July 18	2.21				
July 23		2.02			
July 29	2.21				
Aug. 2				2.76	
Aug. 6			2.21		
Aug. 7		1.84			
Aug. 23	1.84				
TOTAL	8.83	5.88	5.15	2.76	0.00

Table 10--Rainfall records, Fort Collins, Colorado, 1968

Week	Precipitation Inches	Week	Precipitation Inches
May 5 - May 11	0.07	July 14 - July 20	0.10
May 12 - May 18	0.43	July 21 - July 27	0.19
May 19 - May 25	1.92	July 28 - Aug. 3	0.00
May 26 - June 1	0.06	Aug. 4 - Aug. 10	1.05
June 2 - June 8	0.20	Aug. 11 - Aug. 17	1.25
June 9 - June 15	0.12	Aug. 18 - Aug. 24	0.00
June 16 - June 22	0.00	Aug. 25 - Aug. 31	0.00
June 23 - June 29	0.48	Sept. 1 - Sept. 7	0.00
June 30 - July 6	0.02		
July 7 - July 13	0.01	Total for season	5.90

* * *

Table 11--Grain yield in bushels per acre, corn, Fort Collins, Colorado, 1968

Nitrogen fertilizer treatments (lbs./acre)	Irrigation treatments I	II	III	IV	V
			Bushels/acre		
200	1/ 143.7		141.9		96.2
150		136.7		115.1	
100	140.4		128.7		101.4
50		150.3		100.0	
0	126.2		116.5		82.0

1/ One bushel = 56 lbs. One lb. = 0.45 kg.

capacity (FC), and soil moisture tension of 15 bars equivalent to 12.4 percent of soil moisture was considered as permanent wilting point (PWP). The levels of gravimetric soil moisture (percent), and volumetric soil moisture (percent), (averaged over four soil layers, of one foot each) corresponding to selected values of soil moisture tension over the relevant range of moisture situations are presented in table 12.

Table 12--Soil water relationships, Fort Collins Agronomy Research Station

Moisture tension, bars 1/	1/3	1/2	1	5	10	15
Gravimetric soil moisture, percent 2/	28.2	26.1	19.2	14.6	12.9	12.4
Volumetric soil moisture, percent 2/	38.1	35.2	25.9	19.7	17.4	16.7

1/ Desorption data from pressure membrane.
2/ Averaged over four layers of one foot each.

More details on the experiment can be found in Technical Bulletin 107, Colorado State University Experiment Station, Fort Collins, Colorado, January, 1970.

On the basis of soil moisture tension measurements, tension - soil moisture relationships (table 12), irrigation and rainfall data (tables 9 and 10), soil moisture values (averaged over the two upper soil layers of one foot each) were computed, and the corresponding soil moisture fluctuation curves were drawn (figures A12-A16)5/, and the number of days with soil moisture below 45 percent of AMS were counted for each treatment. The results of the number

5/ Two layer averages were computed since the moisture variation in the third foot layer was very small. It was at field capacity throughout the season in treatments I, II and III, and only slightly below field capacity during the last part of the season (August) in treatments IV and V.

of days below 45 percent AMS along with the nitrogen fertilizer and yield data (the latter two transformed into the metric system for sake of conformity with the previous section) are shown in table 13.

The Estimated Functions

The following response functions were specified and estimated with reference to the data in table 13:

$$(9)\ \hat{y} = 9{,}195(\underset{**}{.9905})^{x_1}\ (\underset{**}{.9993})^{(225-x_2)} \qquad R^2 = 0.90$$

$$(10)\ \hat{y} = 10{,}000(.9899)^{x_1}\ (1 - .16156e^{-1.0867x_2 .10^{-3}}) \qquad \bar{R} = 0.81$$

with

$\hat{y}$ = estimated corn grain yield kg/ha;

x_1 = number of days with soil moisture below 45 percent AMS in two upper feet of soil;

x_2 = nitrogen fertilizer level kg/ha

** - denotes significance of the parameter at 1 percent probability level.

The asymptotic yield in (10) was imposed to be 10,000 kg/ha. $\bar{R}^2$ was computed as $\Sigma(y_i - \hat{y}_i)^2/\Sigma(y_i - \bar{y}_i)^2$.

Table 14 shows the actual (y_i) and the estimated yield ($\hat{y}_i$) for the various treatments using estimate (10), and the deviations between them.

According to both estimates (9) and (10) each "critical day," namely with soil moisture below 0.45 AMS, during the July-August period reduced the yield by one percent. The asymptotic yield was 9,200 kg/ha according to (9). In (10) it was imposed to equal 10,000.

It is unfortunate that the data available from the 1968 and the 1972 experiments were different and no common basis for the comparison of the results could be designed.

Table 13--Number of days with soil moisture below 45 percent of AMS during July-August season, the level of nitrogen fertilizer and the yield of corn grain, Fort Collins, Colorado, 1968[1/]

Observation number	Irrigation treatment	Days with soil moisture below 45% AMS	Nitrogen fertilizer level, kg/ha	Corn grain yield, kg/ha
1	I	0	0	7,951
2		0	112	8,845
3		0	225	9,053
4	II	0	56	9,469
5		0	169	8,612
6	III	1	0	7,340
7		1	112	8,108
8		1	225	8,940
9	IV	14	56	6,300
10		14	169	7,252
11	V	39	0	5,166
12		39	112	6,388
13		39	225	6,061

1/ The data in tables 5 and 8 were transformed into the metric units using the following relationships: one bushel of corn = 56 lbs; one lb. = 0.45 kg.

Table 14--Actual and estimated values of corn yield using estimate (10), Fort Collins, Colorado 1968 experiment

Observation number	Variables' values[1/] x_1	x_2	y_i	$\hat{y}_i$	$y_i - \hat{y}_i$	Relative deviation Percent
1	0	0	7,951	8,384	-433	- 5
2	0	112	8,845	8,570	275	3
3	0	225	9,053	8,735	318	4
4	0	56	9,469	8,480	989	10
5	0	169	8,612	8,655	- 43	0
6	1	0	7,340	8,300	-960	-13
7	1	112	8,108	8,484	-376	- 5
8	1	225	8,940	8,646	294	3
9	14	56	6,300	7,148	-848	-13
10	14	169	7,252	6,781	471	6
11	39	0	5,166	5,640	474	9
12	39	112	6,388	5,765	623	10
13	39	225	6,061	5,876	185	3

1/ See text for the definition of variables.

Two major differences were: In the 1972 experiment only averaged soil moisture measurements to the depth of 195 cm (78 inches) were available, while for the 1968 experiment there were observations only for three top layers of one foot each (91.5 cm). 6/ There was no detailed information on the silking period in 1968, while it was well defined in 1972. In the analysis of the 1972 experiment the number of critical days during the silking period was found to be an important factor.

Summary and Conclusions

In this study empirical estimates of response functions of corn to soil moisture were presented.

The analyses provide evidence that the concept of "critical days" as defined in this study, provides a useful basis for the expression of soil moisture variables in the specification of crop response functions. Under conditions with no extreme weather situations, the reference to soil moisture alone, rather than to a combination of soil moisture and other atmospheric evaporative conditions, is sufficient for the definition of a "critical day," for sake of a statistical analysis and estimation of response functions. Obviously, such a definition would not be satisfactory from the point of view of plant physiologists.

The concept of "critical days" has an advantage of having an operation implication in irrigation management and scheduling. The simplest rule is to irrigate crops before soil moisture falls to the level that would allow a "critical day" to occur, because the occurrence of a "critical day" reduces the yield of the crop. These studies show, however, that at some periods

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6/ This explains the different levels of critical soil moisture in the two years (.55 AMS in 1972 and .45 AMS in 1968) which were found appropriate from the point of view of statistical fit.

during the growing season a "critical day" will result in a greater decline in yield than other periods. For instance, during the silking, tasseling period for corn each critical day reduces yields an estimated 2 to 2.5 percent while critical days before or after reduce yields .75 percent to 1 percent per day. If it is not possible to irrigate to avoid critical days on all crops, then one should apply the marginal principle and allocate water to those crops in which highest loss would occur due to delay in irrigation. Obviously, a precondition for proper management and timing of irrigation is the knowledge of the variations in soil moisture in the irrigated plots during the season. Methods for relatively easy tracing of these variations should be devised and adapted to the farmers' needs, so that the decline in soil moisture can be followed and irrigations scheduled to avoid the development of soil moisture conditions that cause occurrence of "critical days."

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APPENDIX A

Table A1--Actual and estimated values of corn yield using estimate No. 2, Fort Collins, Colorado 1972 irrigation experiment

Treatment	Variables values 1/					
	x_1	x_2	y_i	$\hat{y}_i$	$(y_i - \hat{y}_i)$	Relative deviation 2/
						Percent
1A	.27	6	7,128	7,059	68	1
3AI	.42	7	9,450	7,990	1,460	15
4A	.65	0	10,130	9,720	410	4
1B	.36	5	7,142	7,959	- 817	-11
3B	.47	2	9,037	8,967	70	1
4B	.84	0	9,972	9,872	100	1
1C	.38	9	6,817	7,511	- 694	-10
3C	1.00	5	8,906	9,049	- 143	- 2
4C	1.00	0	9,553	9,917	- 364	- 4
5	1.00	0	9,899	9,917	- 118	- 1

1/ See text for the definition of the variables.

2/ Computed as $\frac{(y_i - \hat{y}_i)}{y_i} \times 100$.

Table A2--Marginal product of x_1^* for selected combinations of x_1^* and x_2, based on Estimate 2

Variables' values[1/] x_1	x_2	$\hat{y}$	$MP_{x_1^*}$ 2/	Relative marginal product 3/ Percent
.30	0	8,211	10	10
.30	5	7,492	92	9.2
.30	10	6,836	84	8.4
.50	0	9,405	32	3.2
.50	5	8,581	29	2.9
.50	10	7,830	26	2.6
.75	0	9,820	7.4	0.74
.75	5	8,960	6.7	0.67
.75	10	8,175	6.1	0.61
1.00	0	9,917	1.72	0.172
1.00	5	9,048	1.57	0.157
1.00	10	8,257	1.43	0.143

1/ See text for the definition of the variables.

2/ Computed according to (3) in text.

3/ Computed as $(MP_{x_1^*}/\hat{A}) \times 100$.

Figure A1-Level of soil moisture in corn irrigation experiment, Fort Collins, Colorado, 1972.
Treatment A (one irrigation, July 25)

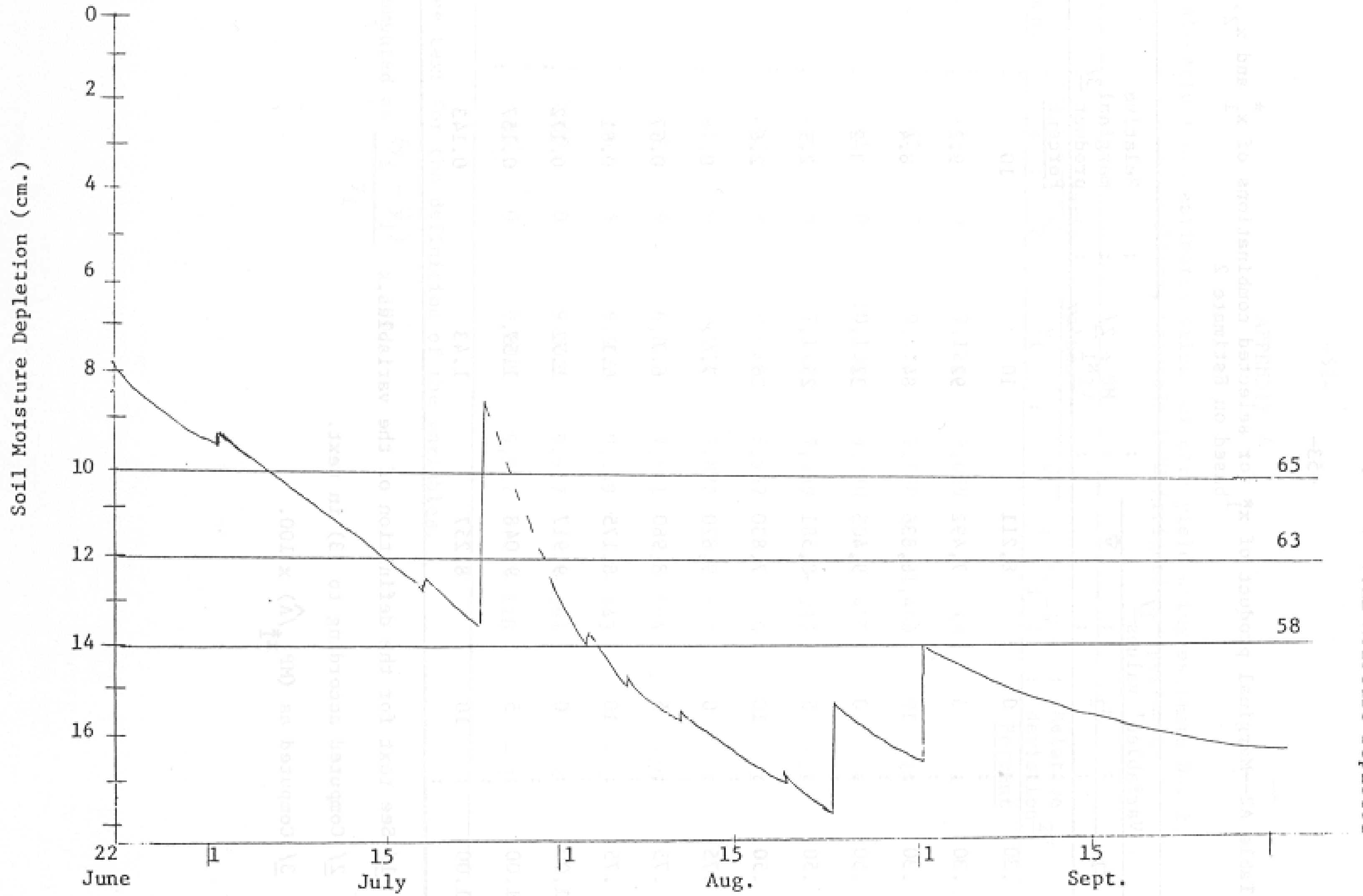


Figure A2--Level of soil moisture in corn irrigation experiment, Fort Collins, Colorado, 1972. Treatment 3A (3 irrigations: July 13, July 25, Aug. 8)

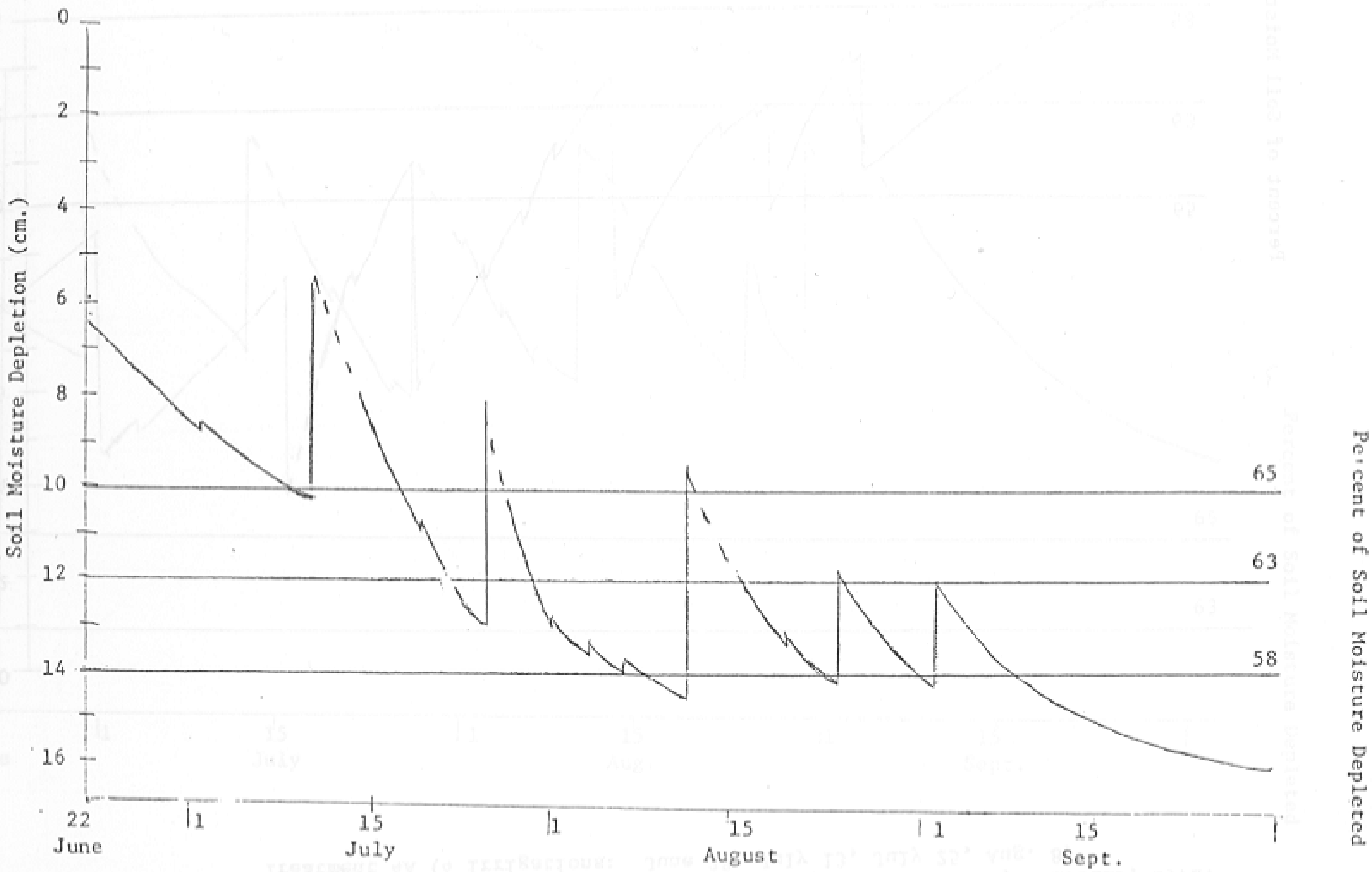


Figure A3--Level of soil moisture in corn irrigation experiment, Fort Collins, Colorado, 1972.
Treatment 4A (4 irrigations: June 28, July 13, July 25, Aug. 8)

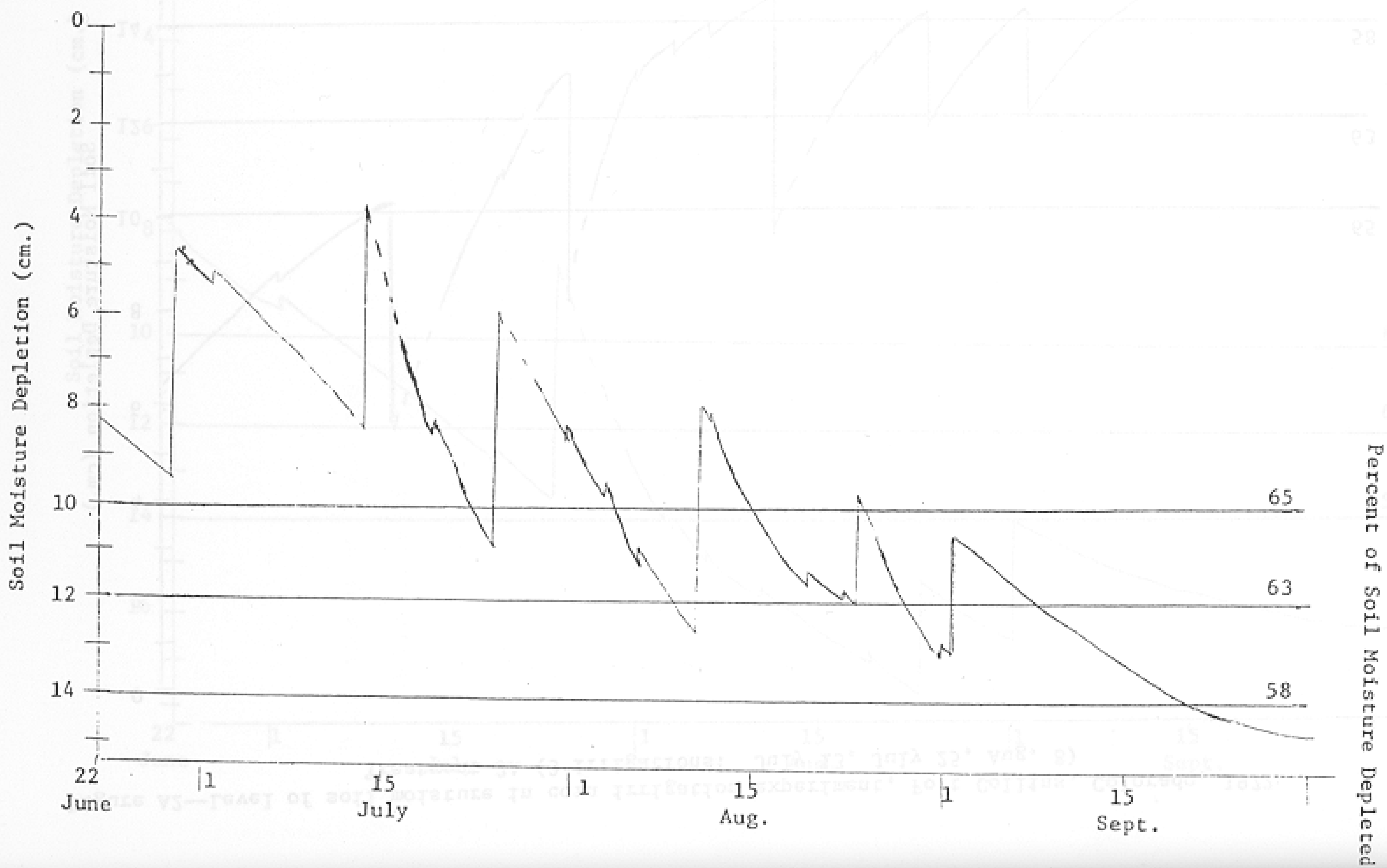


Figure A4--Level of soil moisture in corn irrigation experiment, Fort Collins, Colorado, 1972.
Treatment 5 (5 irrigations: June 28, July 13, July 28, Aug. 8, Aug. 28)

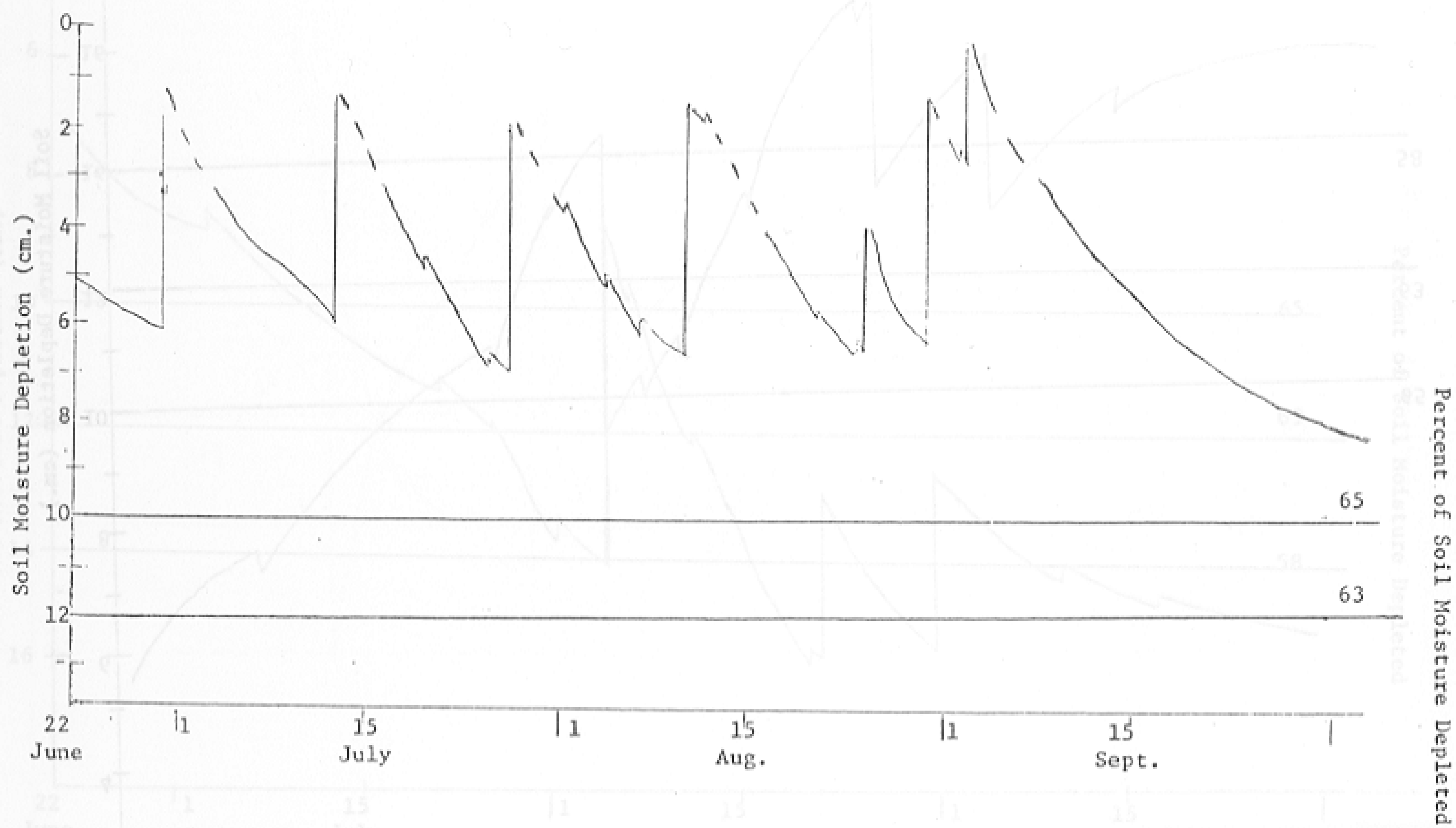


Figure A5-Level of soil moisture in corn experiment, Ft. Collins, Colorado, 1972. Treatment 1B (one irrigation July 31)

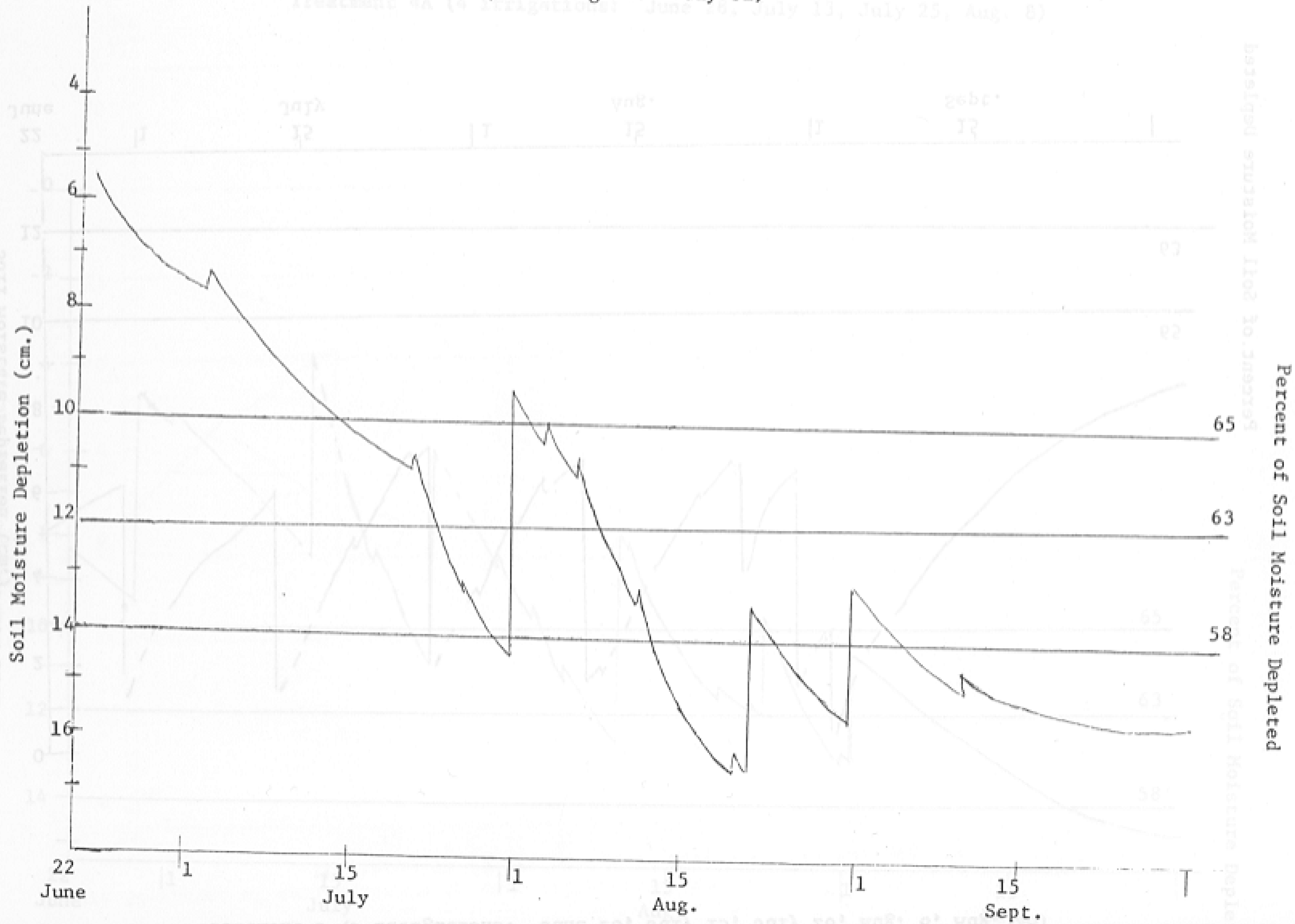


Figure A6 -Level of soil moisture in corn experiment, Fort Collins, Colorado, 1972. Treatment 1C (one irrigation, Aug. 4)

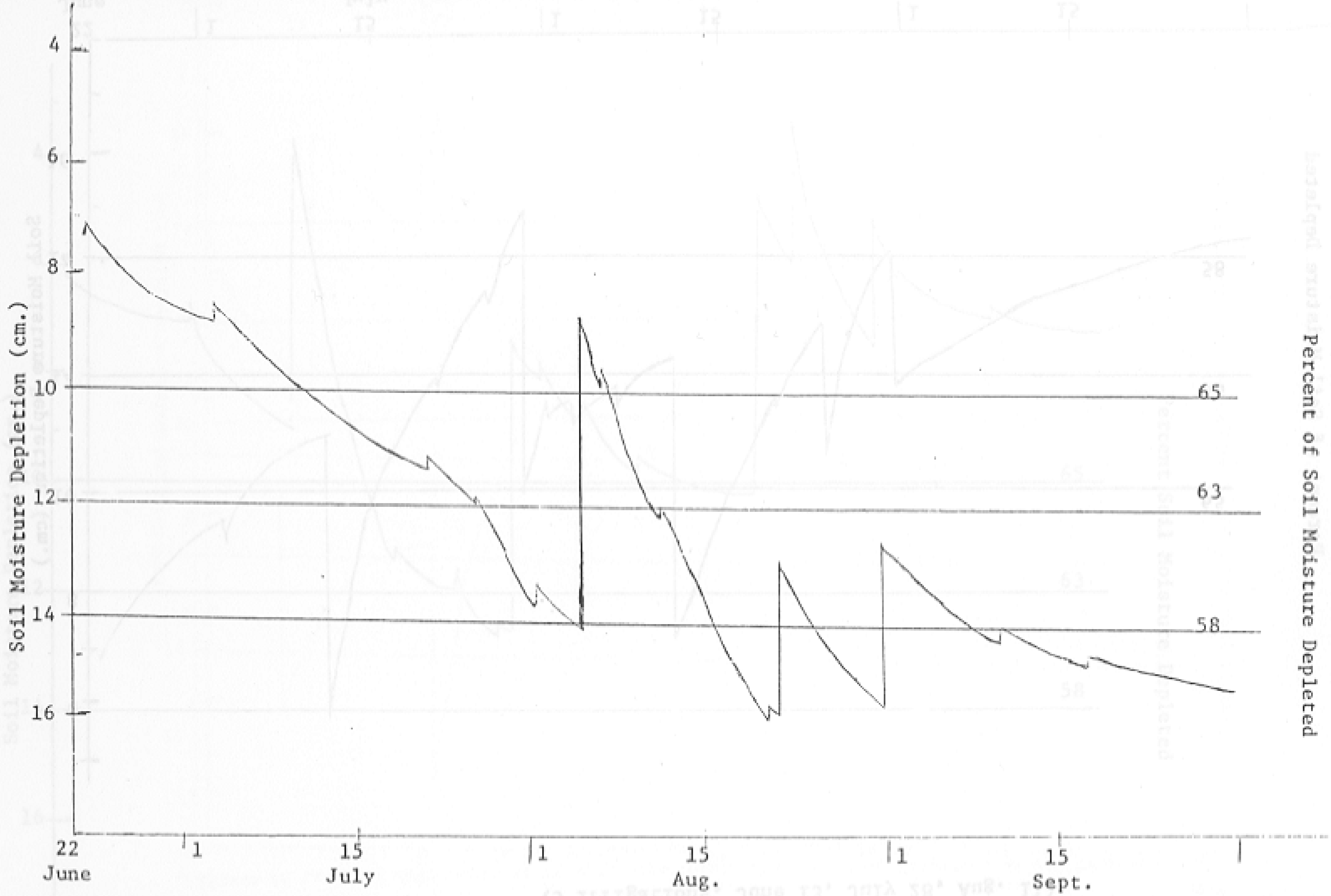


Figure A7-Level of soil moisture in corn experiment, Fort Collins, Colorado, 1972. Treatment 3B
(3 irrigations, June 13, July 28, Aug. 11)

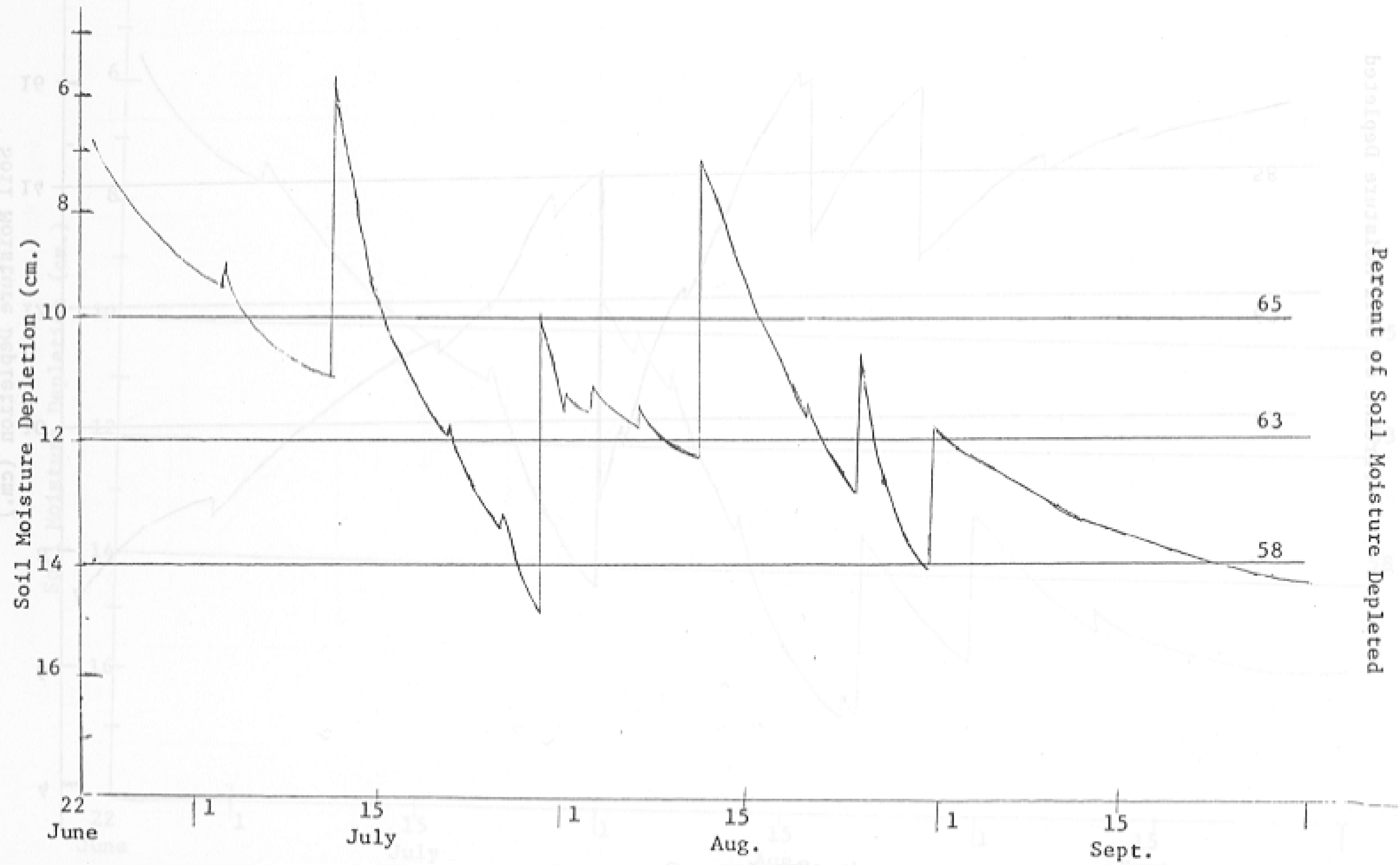


Figure A8—Level of soil moisture in corn experiment, Fort Collins, Colorado, 1972. Treatment 3C
(3 irrigations, June 13, Aug. 2, Aug. 21)

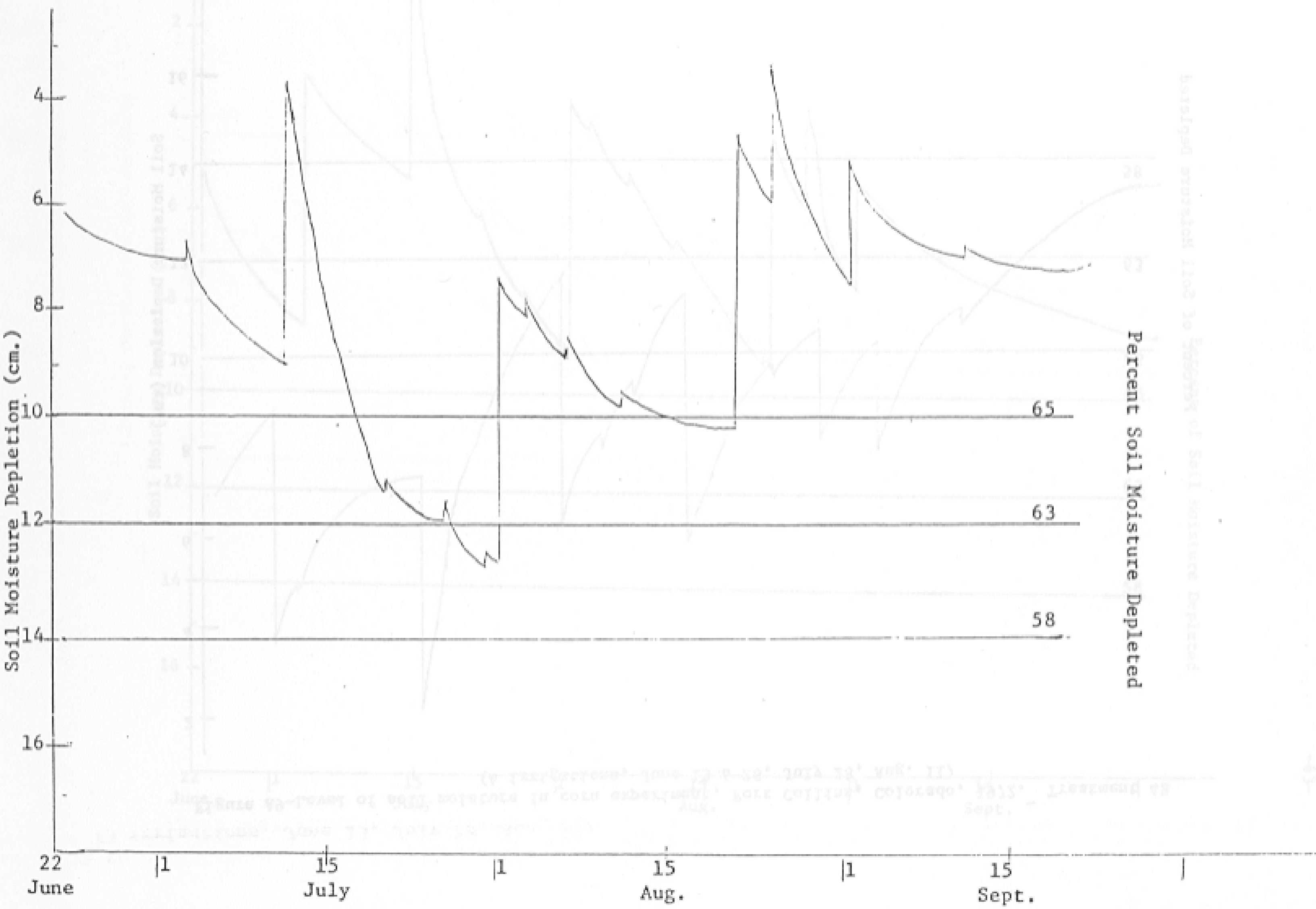


Figure A9—Level of soil moisture in corn experiment, Fort Collins, Colorado, 1972. Treatment 4B
(4 irrigations, June 13 & 28, July 28, Aug. 11)

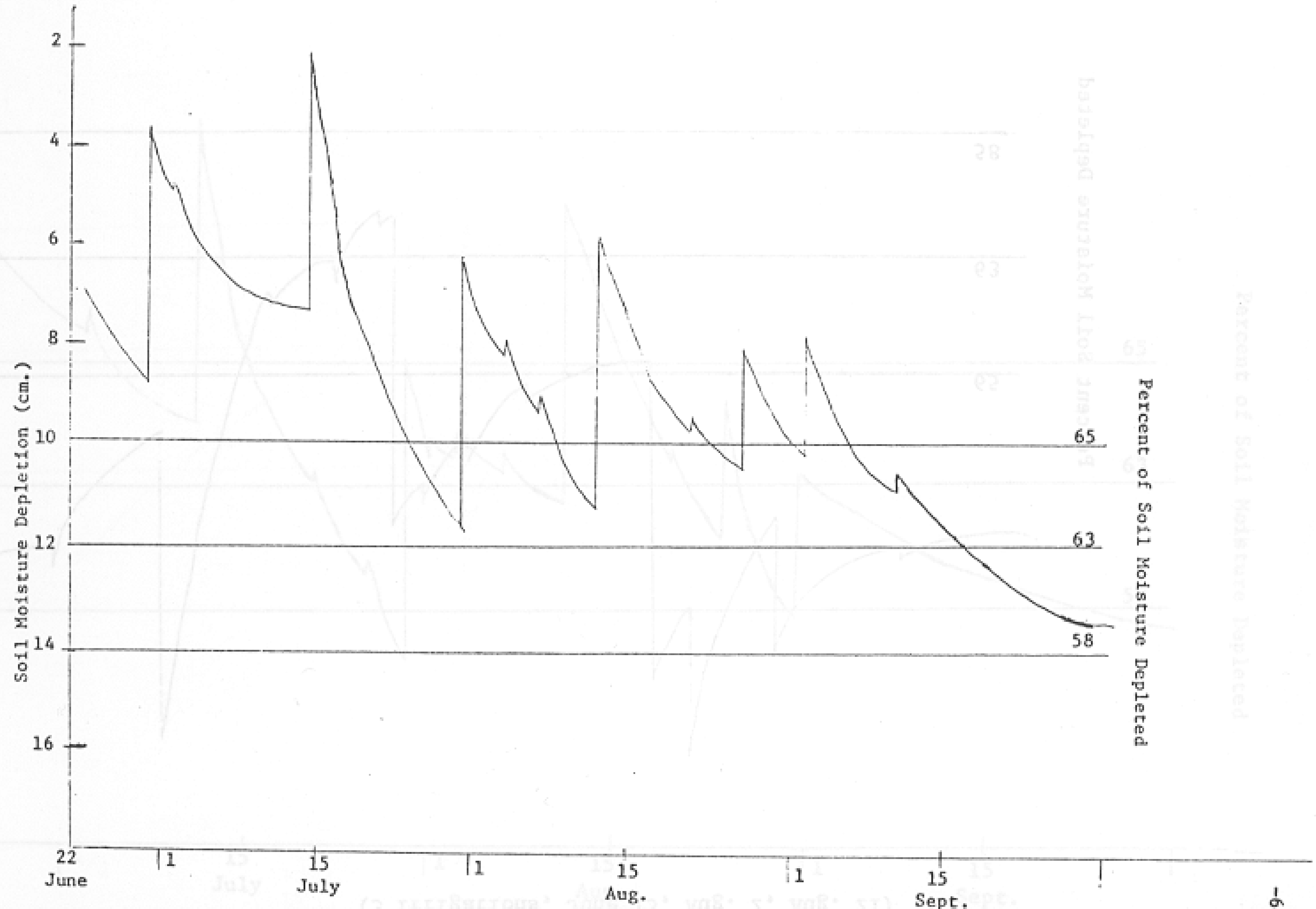


Figure A10.--Level of soil moisture in corn experiment, Fort Collins, Colorado, 1972. Treatment 4C (4 irrigations, June 13 & 28, July 31, Aug. 21)

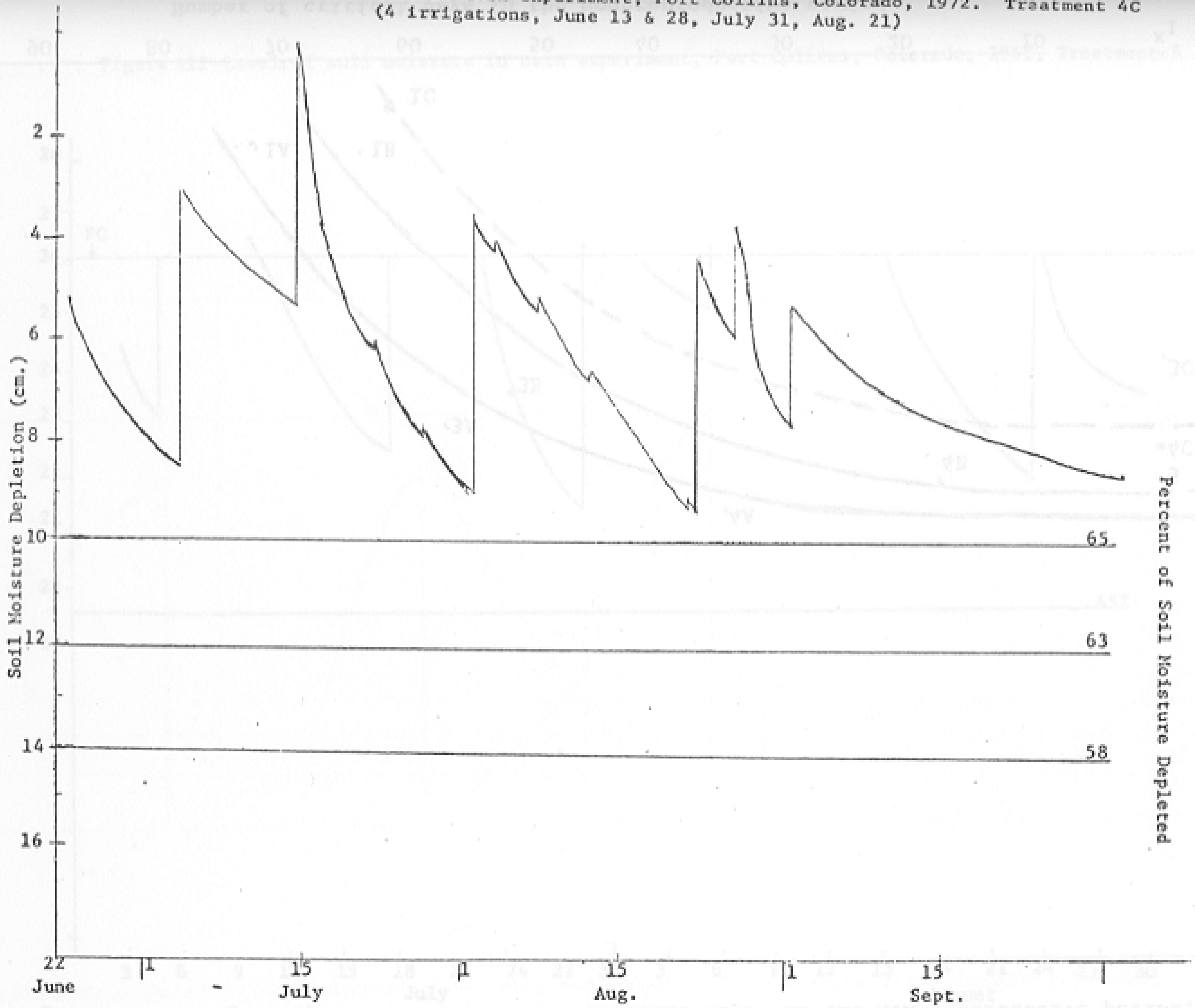


Figure A11--Corn grain yield vs. number of "critical days" in the non-reproductive period, Ft. Collins, 1972

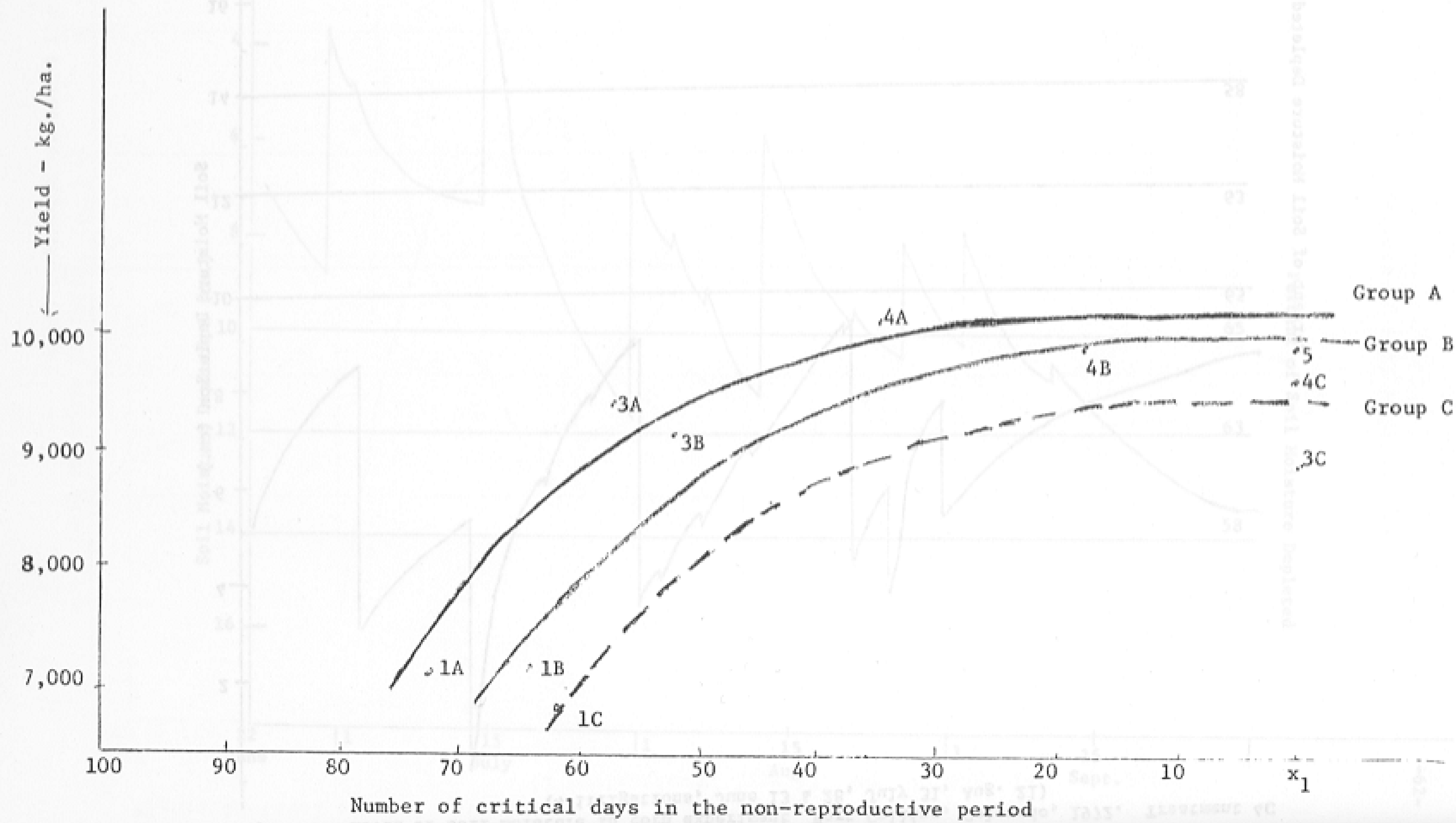


Figure A12 -Level of soil moisture in corn experiment, Fort Collins, Colorado, 1968, Treatment I

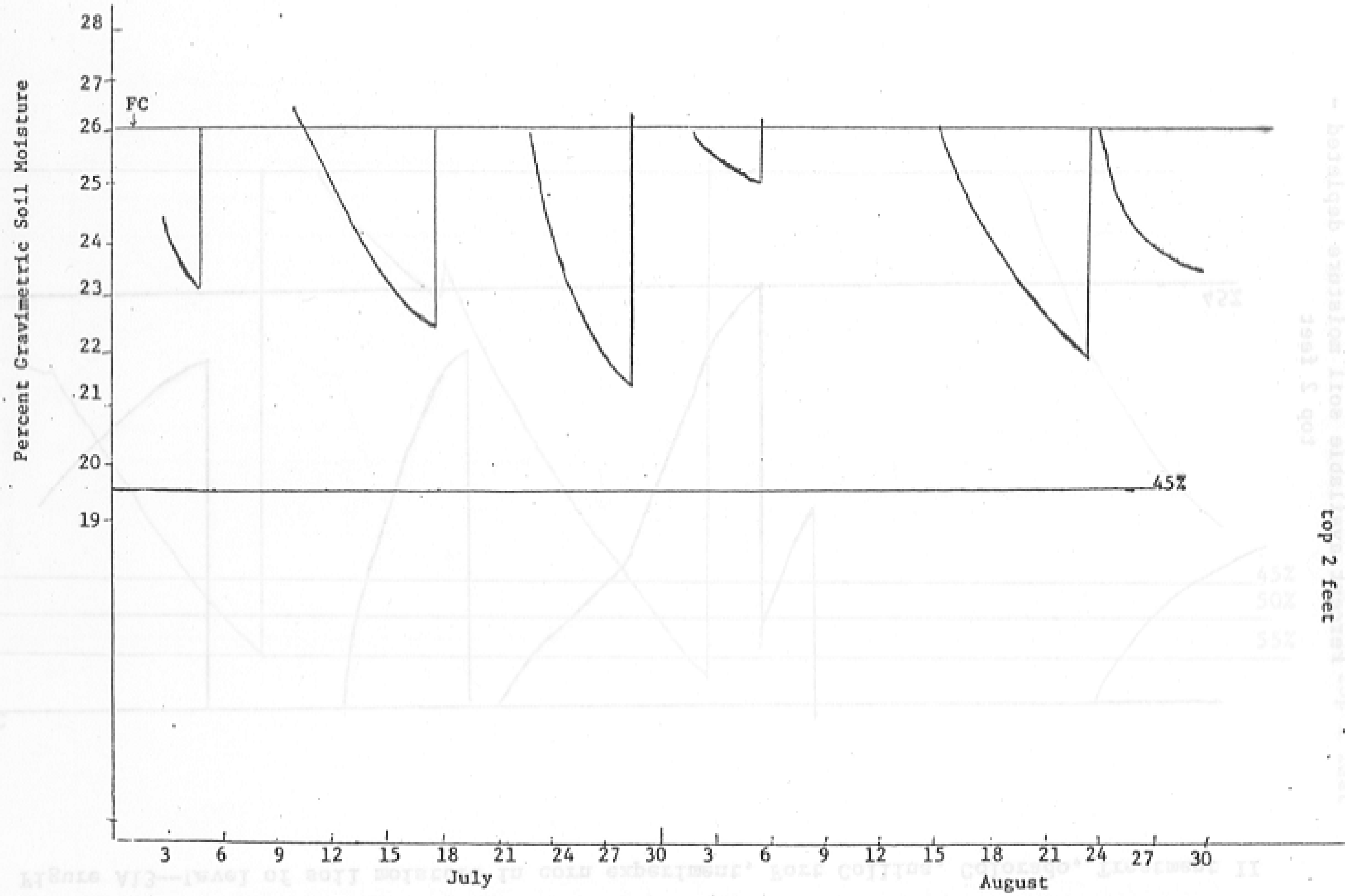


Figure A13--Level of soil moisture in corn experiment, Fort Collins, Colorado, Treatment II

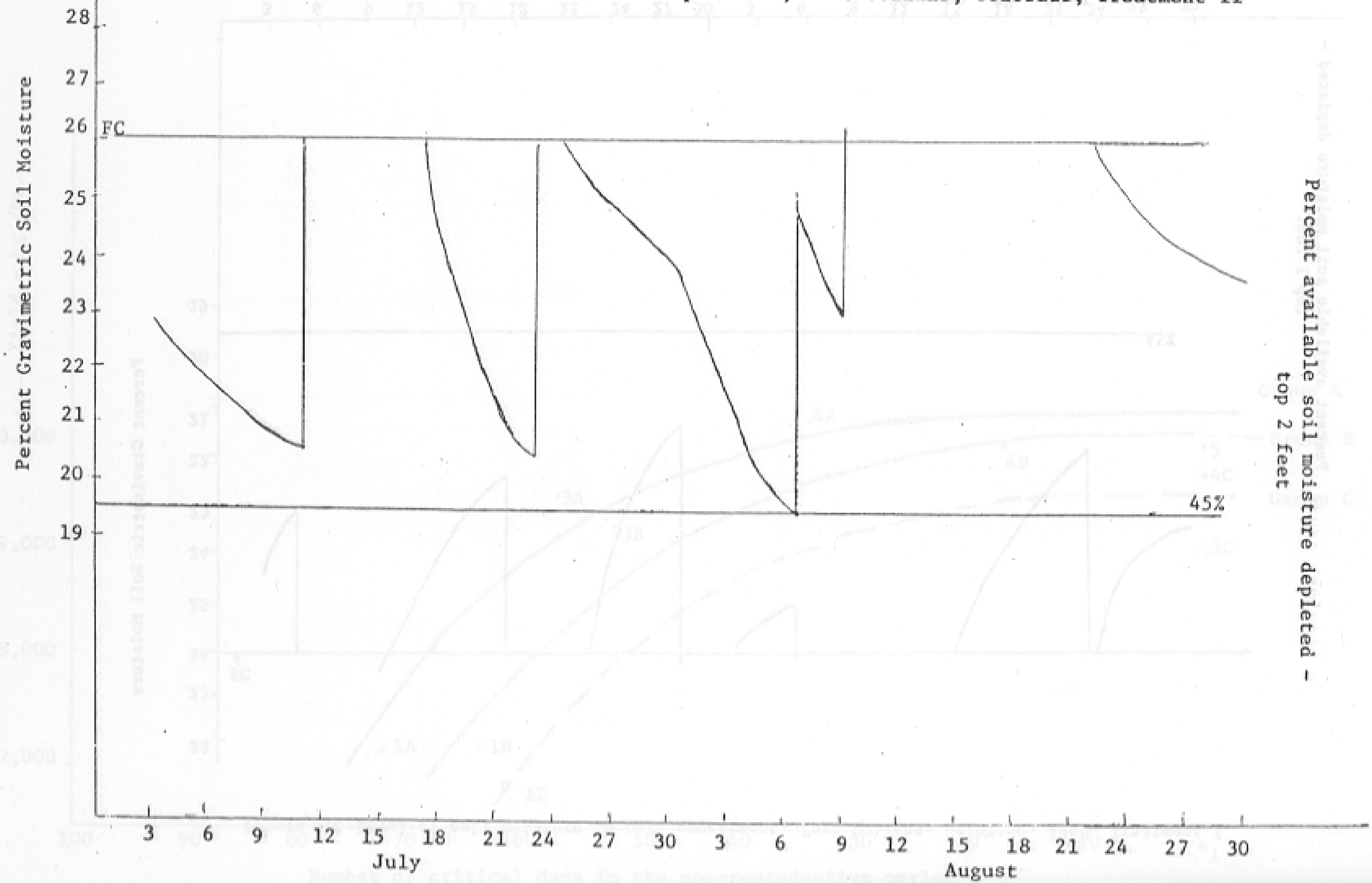


Figure A14--Level of soil moisture in corn experiment, Fort Collins, Colorado, 1968, Treatment III

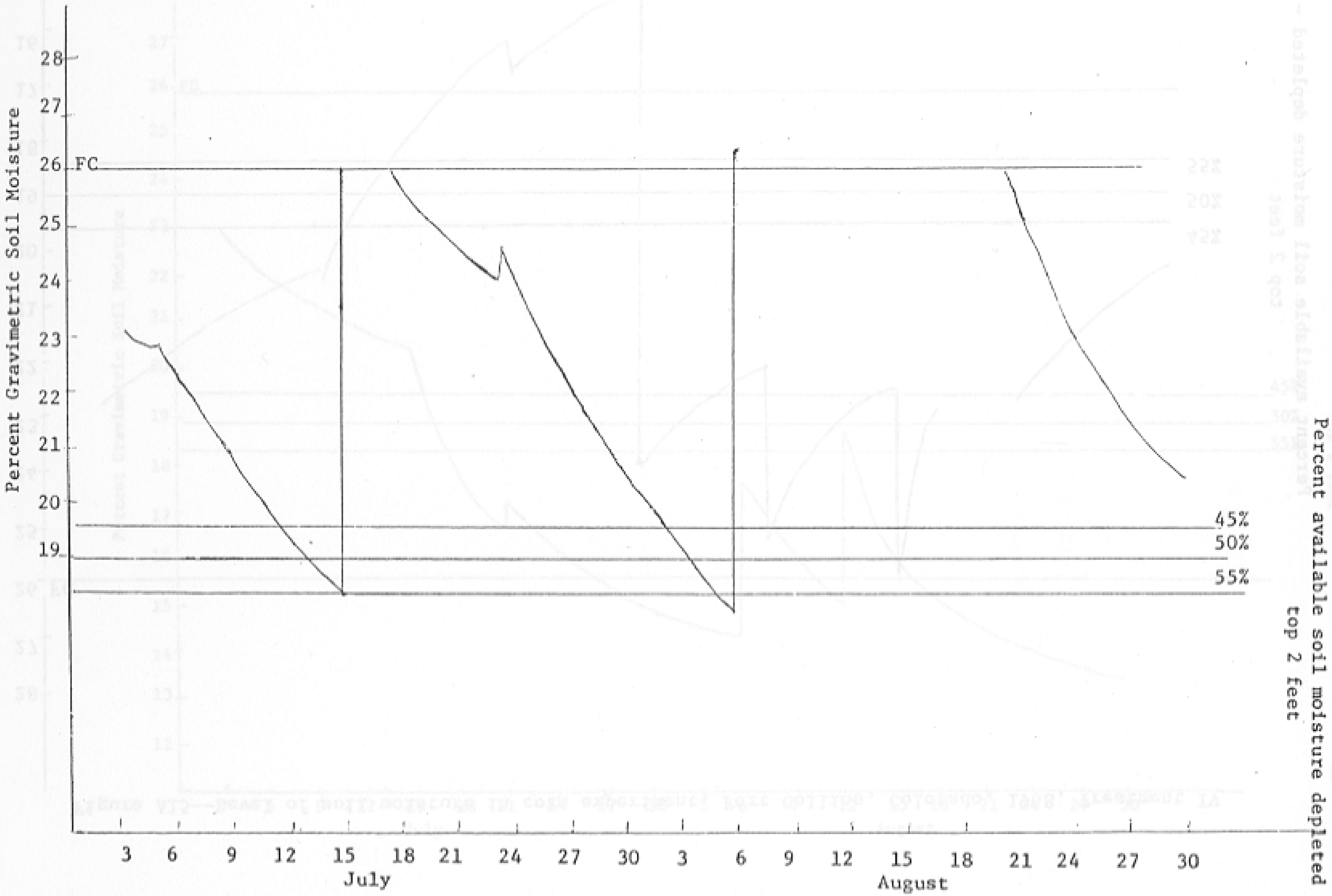


Figure A15--Level of soil moisture in corn experiment, Fort Collins, Colorado, 1968, Treatment IV

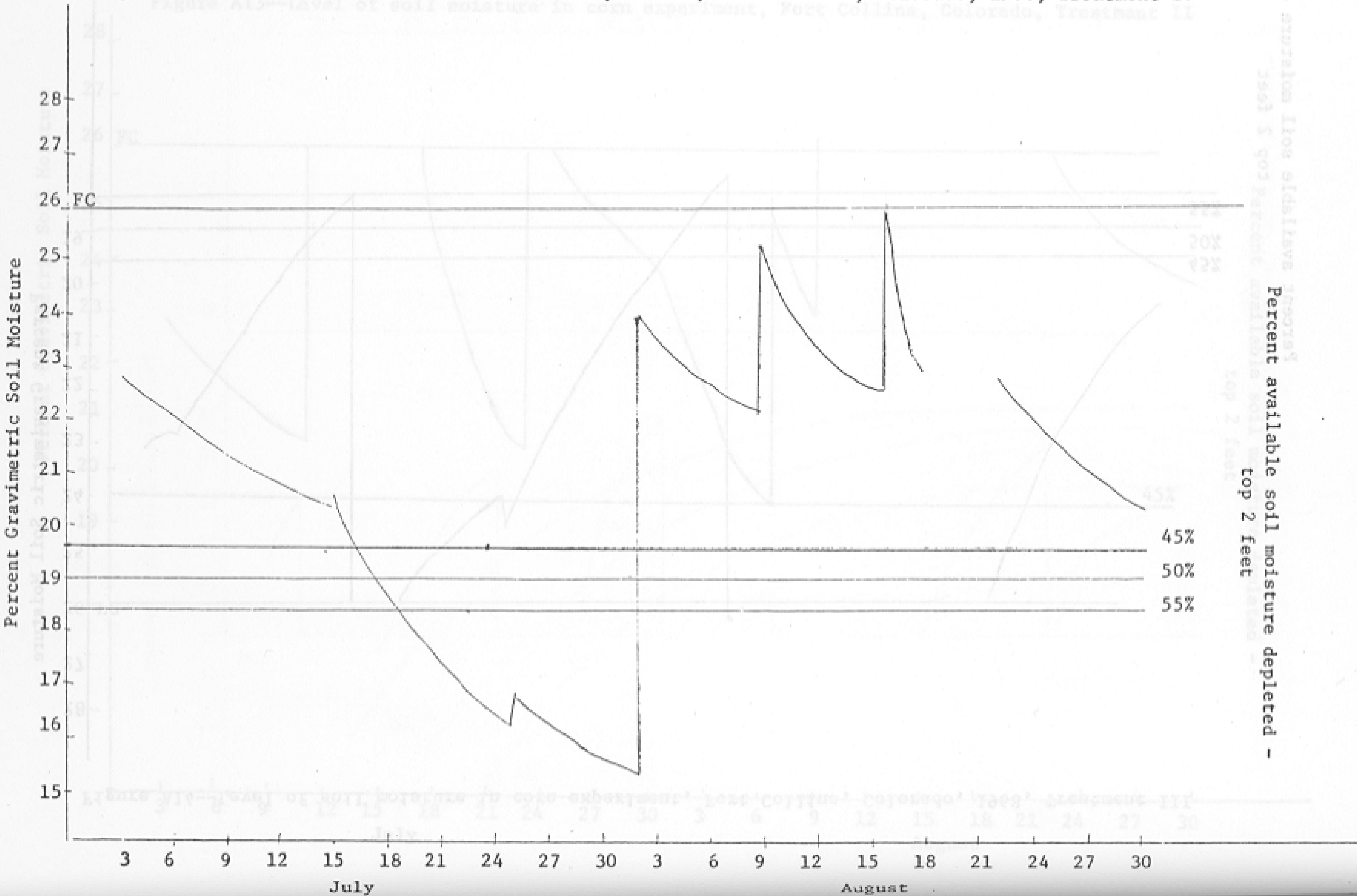
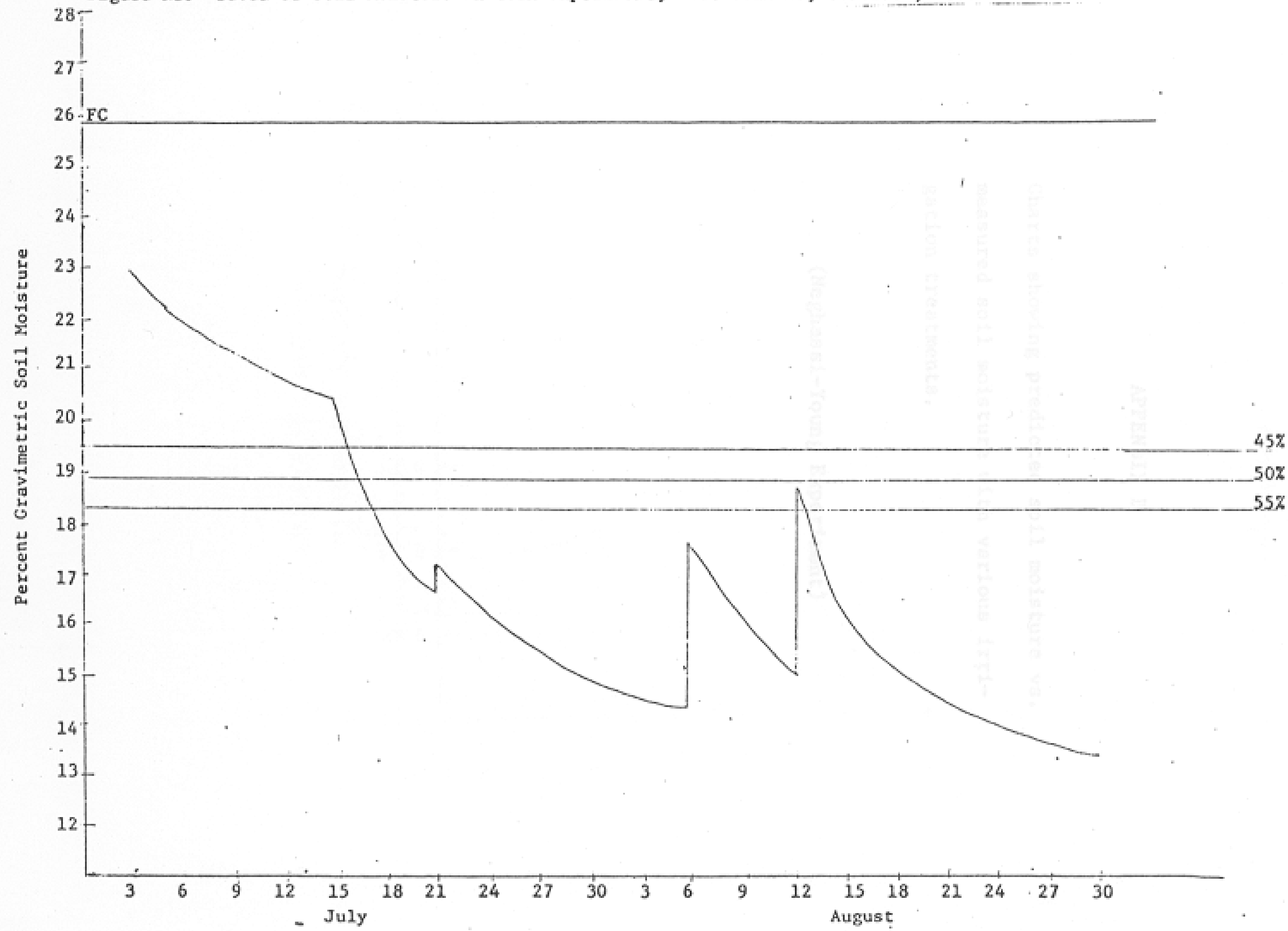


Figure A16--Level of soil moisture in corn experiment, Fort Collins, Colorado, Treatment V



APPENDIX B

Charts showing predicted soil moisture vs. measured soil moisture with various irrigation treatments.

(Neghassi-Young Experiment)

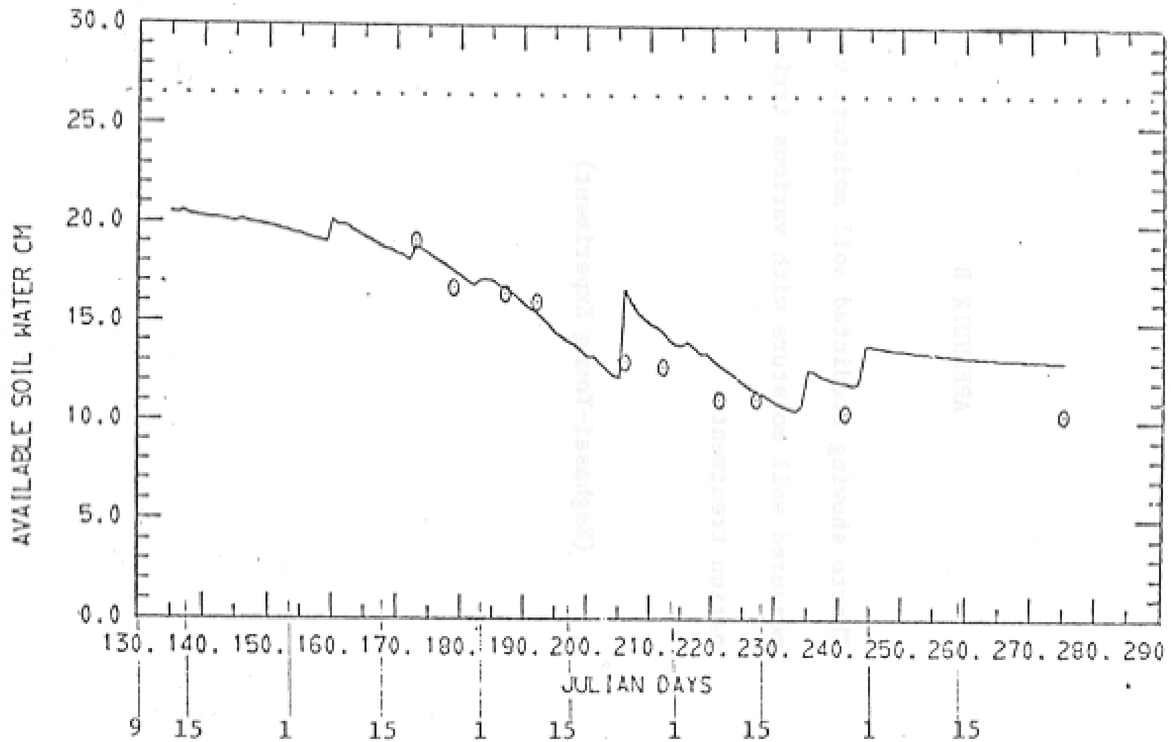


FIG. B1 - TREATMENT 1A-- ONE 5 CM IRRIGATION

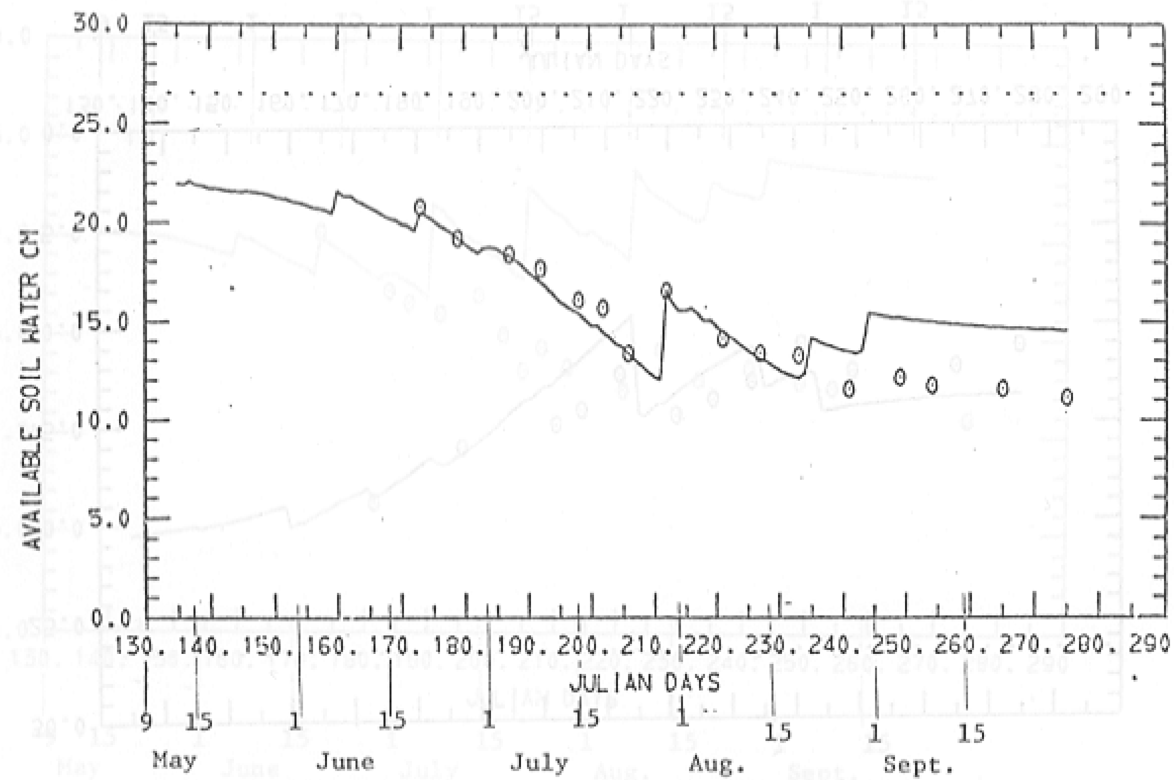


FIG. B2-TREATMENT 1B-- ONE 5 CM IRRIGATION

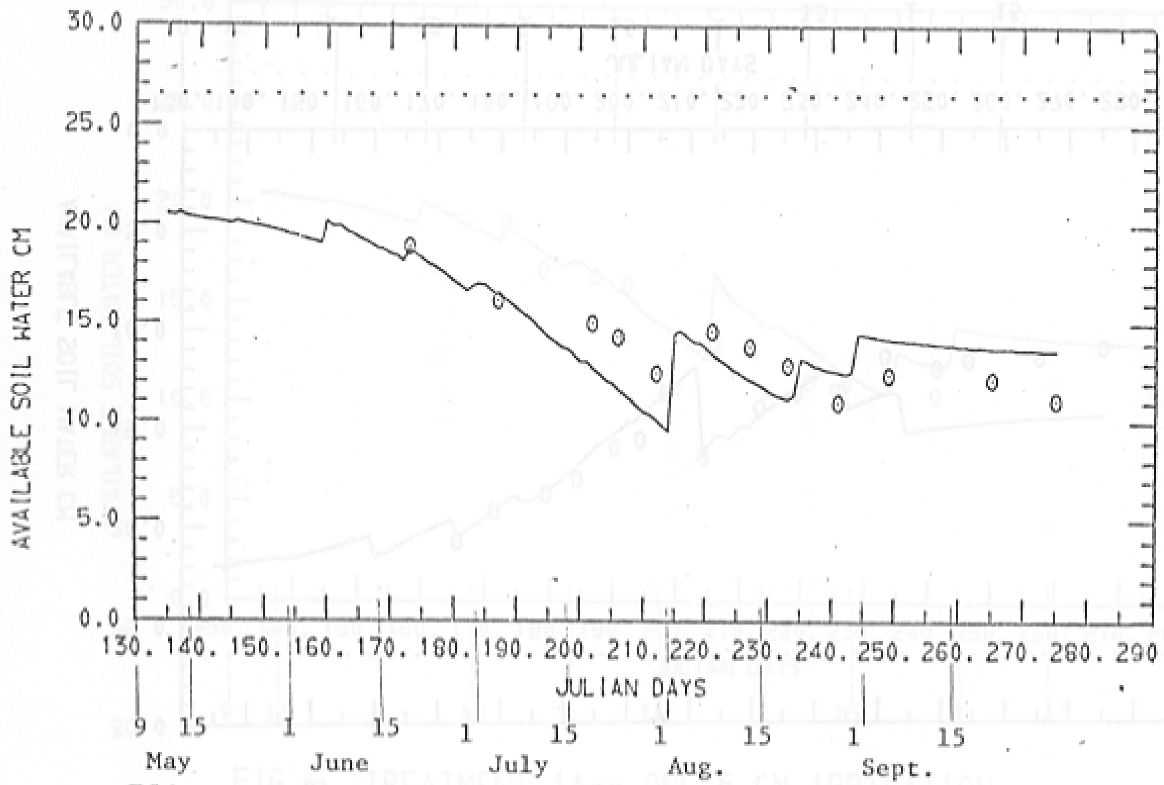


FIG. B3- TREATMENT 1C-- ONE 5 CM IRRIGATION

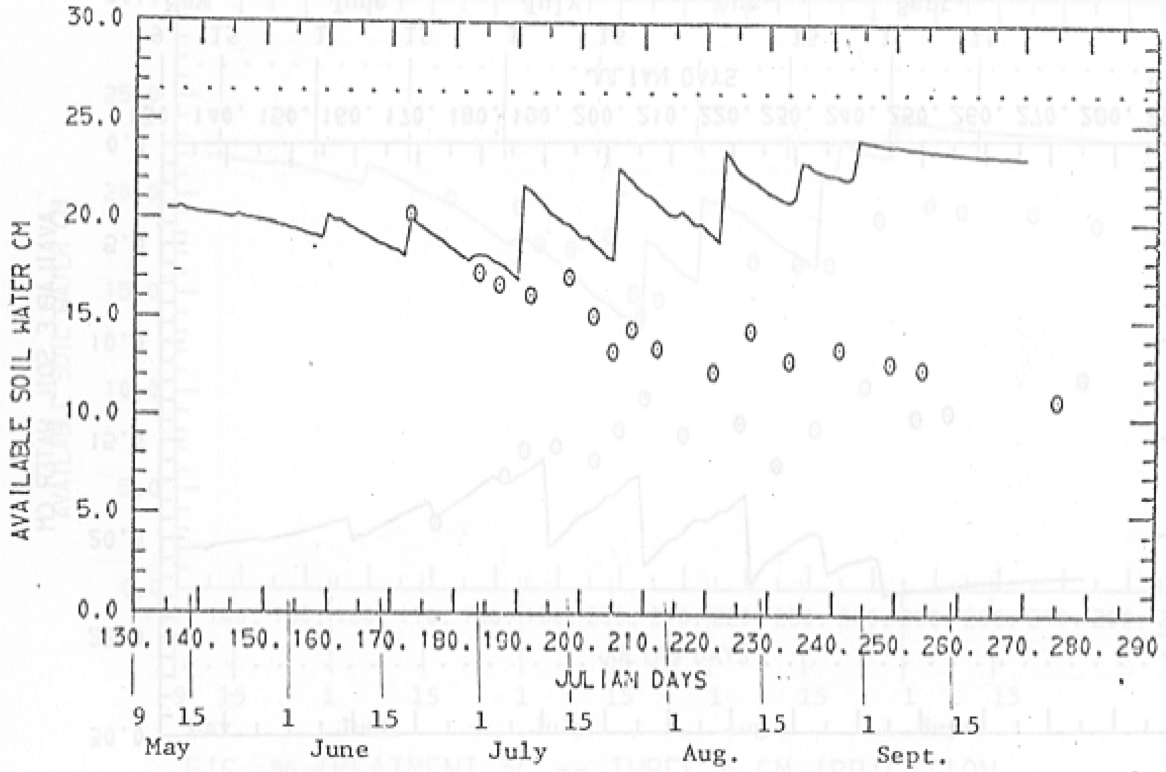


FIG. B4 TREATMENT 3A -- THREE 5 CM IRRIGATION

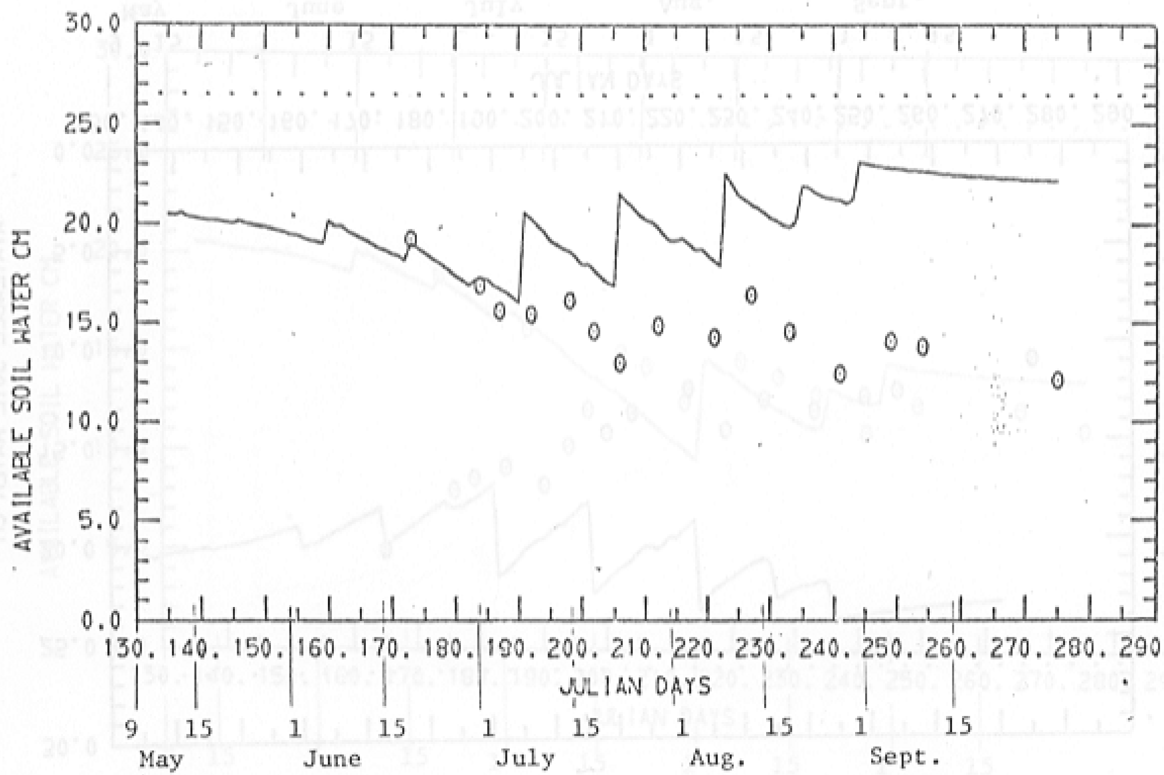


FIG. B5 -TREATMENT 3B -- THREE 5 CM IRRIGATION

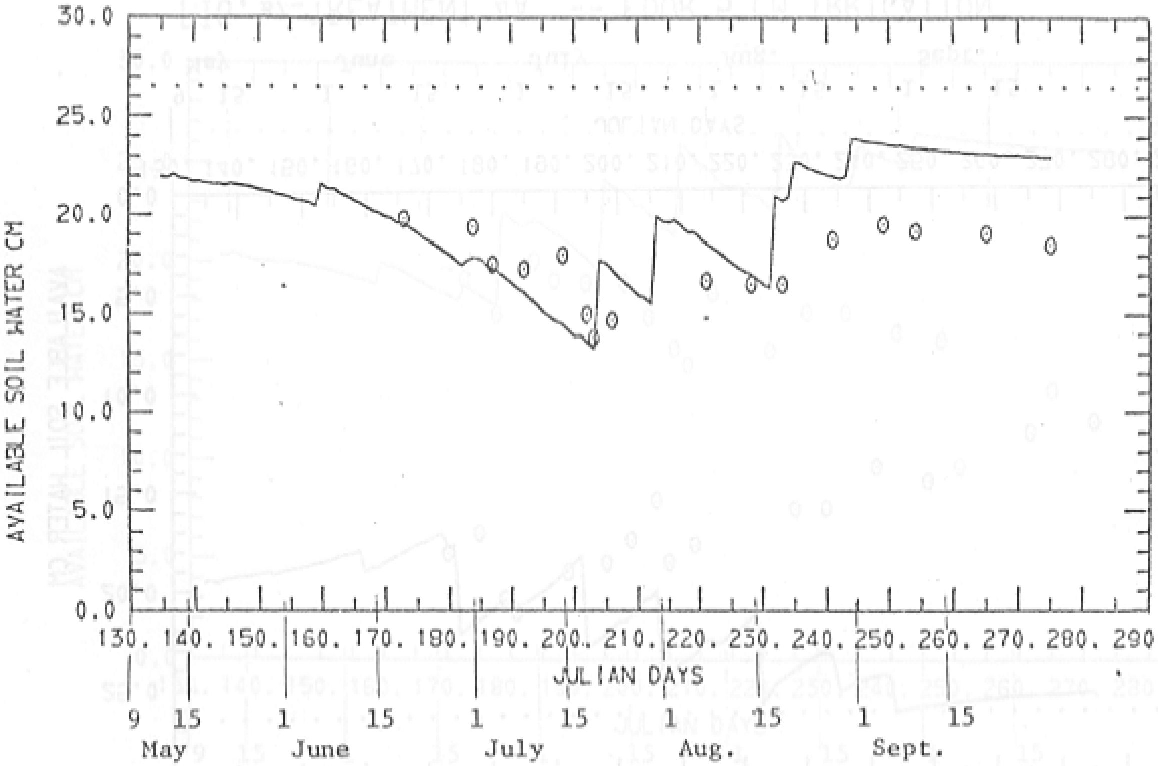


FIG. B6 -TREATMENT 3C -- THREE 5 CM IRRIGATION

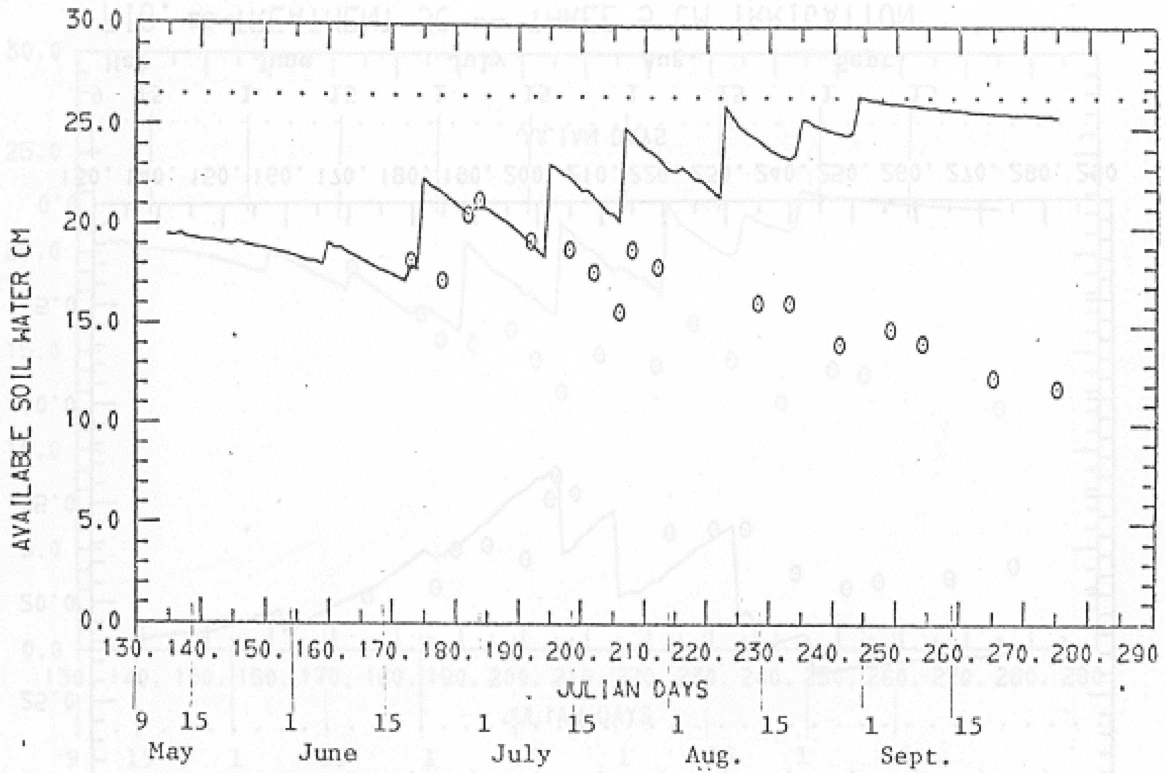


FIG. B7-TREATMENT 4A -- FOUR 5 CM IRRIGATION

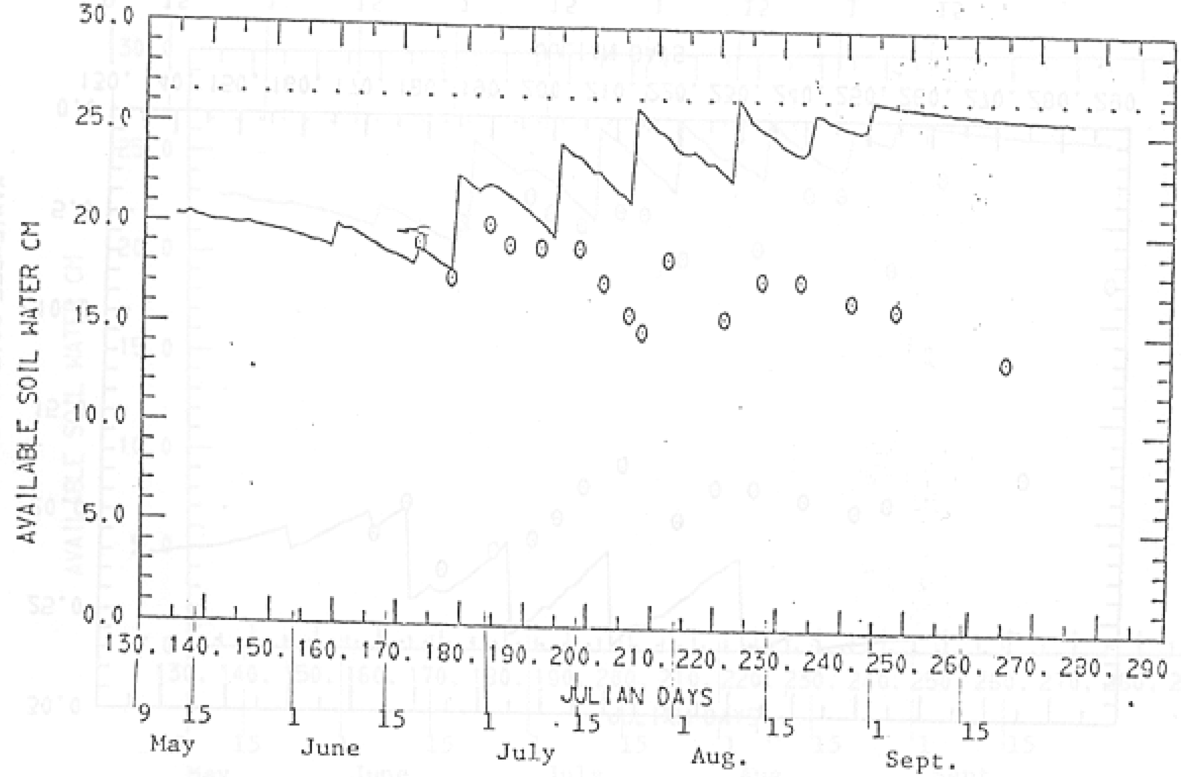


FIG. B8 -TREATMENT 4B -- FOUR 5 CM IRRIGATION

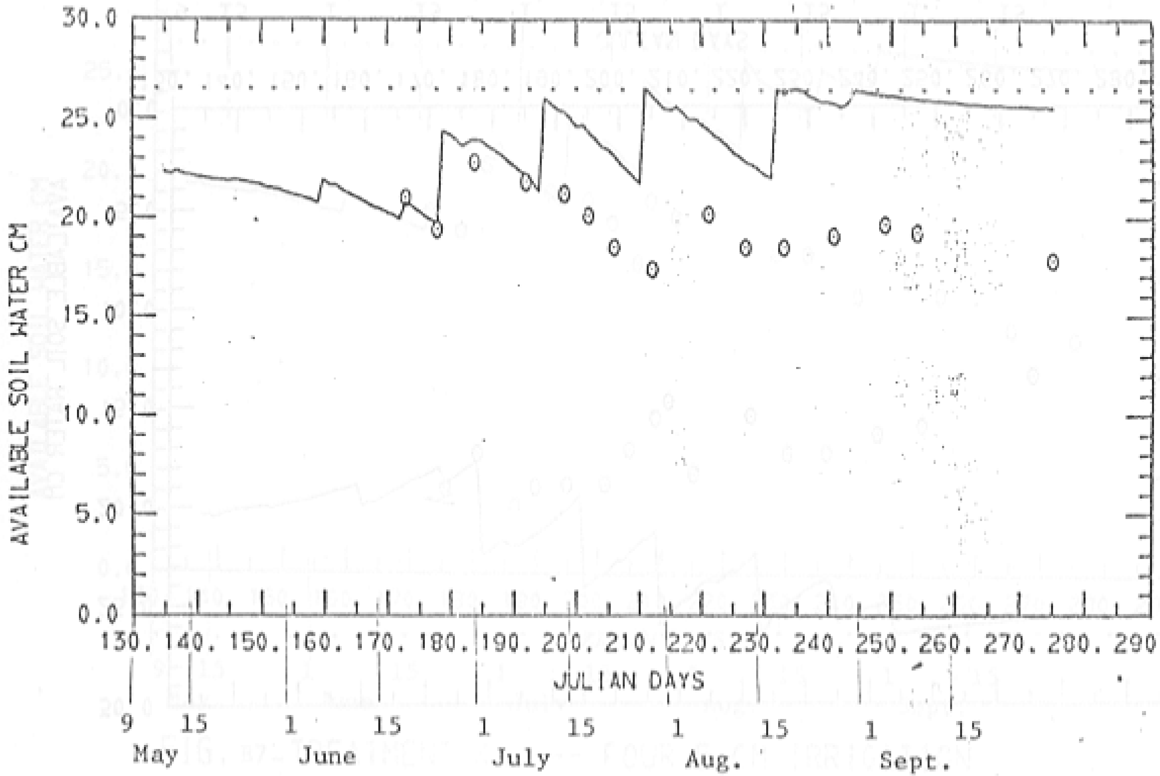


FIG. B9-- TREATMENT 4C -- FOUR 5 CM IRRIGATION

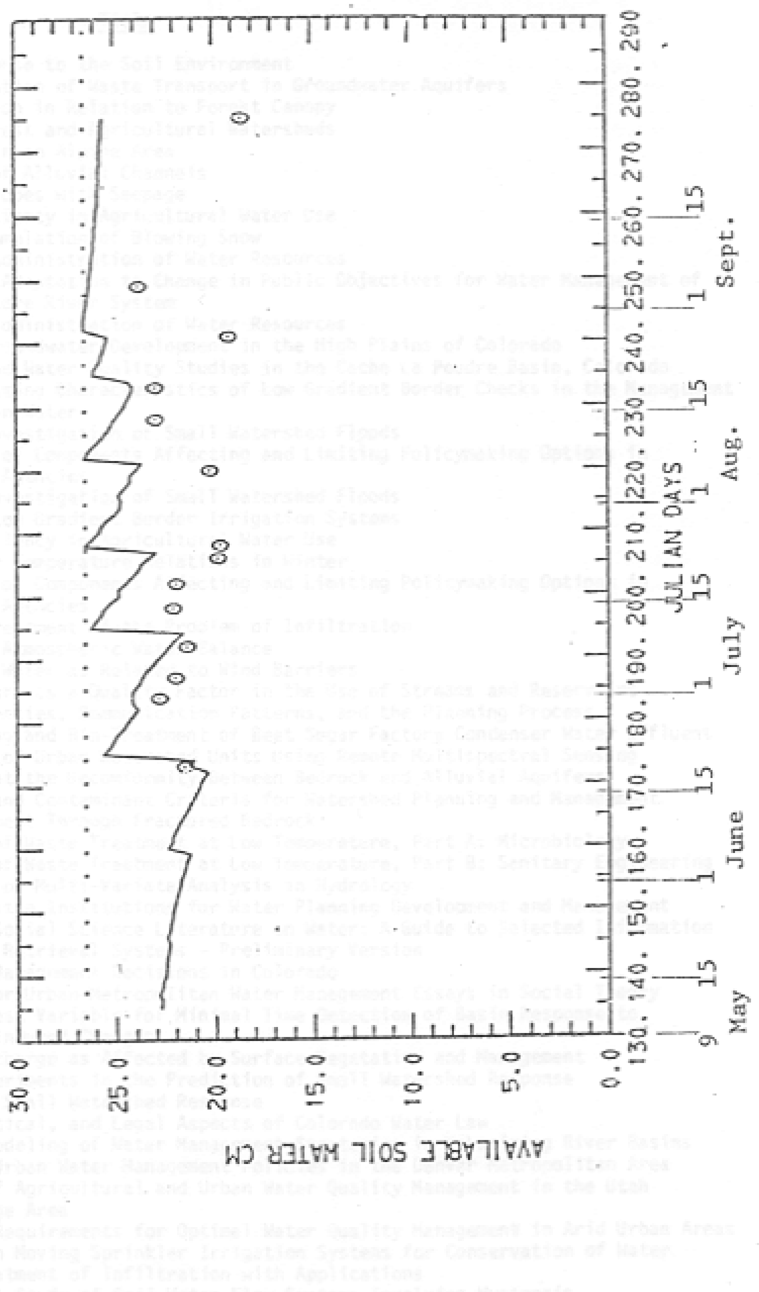


FIG. B10 TREATMENT 5 -- FIVE 5 CM IRRIGATION