CONJUNCTIVE OPERATION OF A SURFACE RESERVOIR AND OF GROUNDWATER STORAGE THROUGH A HYDRAULICALLY CONNECTED STREAM

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ABSTRACT

Analytical solutions are described to represent the impact of net withdrawal from an aquifer on water table elevations and on induced seepage (negative return flow) from a river in hydraulic connection with the aquifer. These analytical solutions are prerequisite to the formulation and solution of a conjunctive optimal strategy of use of surface and ground waters for irrigation purposes. Optimal continuous time solutions are sought for rates of release from an upstream surface reservoir, for diversion rates of streamflows downstream from the dam and upstream of an irrigation area and for rates of pumping in the irrigation zone via the classical techniques of Calculus of Variations.

However instead of leading to an Euler-Lagrange system of partial differential equations the formulation leads to a system of Fredholm linear integral equations of the second kind. The clear economic meaning for the optimal strategy is a trade-off between two marginal costs: immediate value of not incurring a penalty for failing to meet a downstream legal right versus the capitalized cost of additional lift as a result of early pumping in the season.

FOREWORD

Maximum benefit from water use in irrigation is obtained by minimizing the cost of water (assuming cropping practices are fixed). The cost of groundwater is greater than the cost of surface water due to pumping costs. If surface supply is inadequate to meet full water requirements, some groundwater use is necessary. Furthermore, groundwater use may be a mandatory element in an efficient water-cycle system such as occurs in the South Platte River Basin. The management question which this research addresses is, "What is the correct mix of the two sources to optimize returns from the available water?"

A two-pronged approach was used in this study: (1) modify and adapt a hydrologic simulation technology developed with Department of Interior's partial support in a prior matching grant project (CR87) and, (2) develop the theory and procedure for incorporating optimization analysis into the hydrologic model.

The hydrologic system of interest is the South Platte River Basin in Colorado. Water in an alluvial aquifer in good hydraulic connection with the river is managed conjunctively with surface water. Groundwater pumping is permitted only if its impacts on surface stream flow is offset by augmentation water. Water users contemplate a main-stem storage reservoir and need new technology to find the best conjunctive reservoir and groundwater management strategy.

A previously developed hydrologic simulation model was modified to incorporate the presence of an upstream storage reservoir. Possible combinations of storage capacity, release rules for the reservoir and pumping rules for downstream aquifer were investigated with the model. The operational capability of the model to simulate this system on a weekly time scale was demonstrated. Its utility for testing and evaluating conjunctive management options was likewise demonstrated to be excellent.

A dissertation on the conjunctive surface-groundwater simulation model will be available from Colorado State University (Restrepo, 1984) in the future. A table of contents is appended at the end of this report. Technical details for an earlier version of model were previously reported in Completion Report No. 87 available from the Colorado Water Resources Research Institute and are not repeated in this report. More advanced modeling features were developed for the Colorado Commission of Higher Education, the Groundwater Users Association of the South Platte and the Ministry of Agriculture and Water of the Kingdom of Saudi Arabia (Illangasekare and Morel-Seytoux, 1983a,b; Morel-Seytoux and Illangasekare, 1983).

The second approach to meeting water user need for new technology is development of innovative methodology for incorporating optimization capability into the hydrologic model. A theory and procedure for finding an optimal strategy for managing surface storage conjunctively with groundwater pumping has been developed. This report gives details of the theory and the procedure. A hypothetical, idealized case is used to illustrate its application.

The next step in development of this new combined hydrologic simulation-optimization technology will be to incorporate the optimization procedure with the simulation model. With this combination optimal reservoir release and groundwater pumping decisions can be made continuously throughout the season of operation. Of course, the new technology will be equally valuable for initial planning of project operations.

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RESEARCH OBJECTIVES

The overall objective of the research was the development of a methodology to demonstrate the value of conjunctive management of an upstream surface reservoir with a downstream aquifer as water supplies. The methodology must incorporat properly the physical interactions between the stream, the aquifer and the wells as well as account for the agronomic (irrigation) and legal constraints. The methodology must be cost-effective so that it can be used for actual operations by various local groups of water users.

In this report only a brief review of a promising method of attack will be given. Generally speaking the thrust of the research has been in the direction of development of new and imaginative methods that will greatly reduce the cost of management studies of conjunctive use of surface and ground waters when in hydraulic connection without significant reduction in accuracy. In this regard the project was successful.

In a separate document, a dissertation (Restrepo, 1984) a more fully developed classical approach is used to provide specific quantitative answers to problems of management for a reach of the South Platte River. It addresses the problem of finding the optimal capacity and release rules of an upstream reservoir as well as the withdrawal rules for the downstream aquifer storage. A table of contents is appended at the end of this report.

PART I

DRAWDOWN AND RETURN FLOW RESPONSES TO UNIFORM WITHDRAWAL IN A ONE-DIMENSIONAL HOMOGENEOUS SEMI-INFINITE AQUIFER

INITIAL CONDITIONS

At time zero drawdown, s, is zero everywhere (i.e., for $0 \leq x \leq \infty$).

BOUNDARY CONDITIONS

At the river bank of a fully penetrating river drawdown remains zero at all times, i.e., s=0 at x=0 for all times. Abscissa x is measured in a direction perpendicular to river course with origin at river bank.

WITHDRAWAL EXCITATION

Withdrawal occurs uniformly over an interval of length a on each side of the fully penetrating river. The river reach length is L_r. Thus the area of withdrawal (which is also the cultivated irrigated area) is $A = aL_r$. The excitation (withdrawal) rate for the area A is Q (volume per unit time) or the excitation rate per unit area is q (depth per unit time). Naturally Q and q are related by the equation: Q=Aq.

DRAWDOWN RESPONSE TO UNIFORM WITHDRAWAL

The drawdown response due to a unit impulse of uniform withdrawal per unit area over the interval (o,a) satisfying the initial condition of zero drawdown everywhere and zero drawdown at all times at the river bank is easily derived (e.g., Morel-Seytoux, 1977) from the knowledge of the Green's function for the one-dimensional linear Boussinesq equation. The solution is:

$$k_{s,q}(x,t) = \frac{1}{2\phi}\,\{\mathrm{erf}(\frac{a-x}{2\sqrt{\gamma t}}) - \mathrm{erf}(\frac{a+x}{2\sqrt{\gamma t}}) + 2\mathrm{erf}(\frac{x}{2\sqrt{\gamma t}})\} \qquad [1]$$

where ϕ is effective porosity, $\gamma = \frac{T}{\phi}$ is aquifer diffusivity and T is transmissivity.

Verification

For any x in the interval (o,a) at time zero (plus) Eq. [1] yields for drawdown the value:

$$k_{s,q}(x,,o) = \frac{1}{2\phi}\{\mathrm{erf}(\infty) - \mathrm{erf}(\infty) + 2\mathrm{erf}(\infty)\} = \frac{1}{2\phi}[1-1+2] = \frac{1}{\phi}$$

which is correct, representing the instantaneous drawdown to an impulse of withdrawal of one unit volume per unit area. For any $x > a$ at time zero (plus) Eq. [1] yields for drawdown the value:

$$k_{s,q}(x,o) = \frac{1}{2\phi}\{\mathrm{erf}(-\infty) - \mathrm{erf}(\infty) + 2\mathrm{erf}(\infty)\} = \frac{1}{2\phi}[-1-1+2] = 0$$

which is correct since water table is initially horizontal.

For any time at the river bank (x=0) Eq. [1] yields for drawdown the value:

$$k_{s,q}(o,t) = \frac{1}{2\phi}\{\mathrm{erf}(\frac{a}{2\sqrt{\gamma t}}) - \mathrm{erf}(\frac{a}{2\sqrt{\gamma t}}) + 2\mathrm{erf}(o)\} = \frac{1}{\phi}\mathrm{erf}(o) = 0$$

which again checks. Thus Eq. [1] provides correctly the response of drawdown to a uniform unit impulse withdrawal excitation over the interval (o,a).

Drawdown Response to a General Excitation (withdrawal) Rate per Unit Area

The general solution is as usual (Morel-Seytoux, 1979, p.16) of the form:

$$s(x,t) = \int_o^t \frac{1}{2\phi}\{\mathrm{erf}(\frac{a-x}{2\sqrt{\gamma(t-\tau)}}) - \mathrm{erf}(\frac{a+x}{2\sqrt{\gamma(t-\tau)}}) + 2\mathrm{erf}(\frac{x}{2\sqrt{\gamma(t-\tau)}})\}\ q(\tau)d\tau \qquad [2]$$

One obtains the response to the withdrawal discharge $Q(\tau)$ (volume per time) by simply replacing $q(\tau)$ by $\frac{Q(\tau)}{A}$ in Eq. [2] or explicitly:

$$s(x,t) = \frac{1}{2A\phi}\int_o^t\left[\operatorname{erf}\{\frac{a-x}{2\sqrt{\gamma(t-\tau)}}\}-\operatorname{erf}\{\frac{a+x}{2\sqrt{\gamma(t-\tau)}}\}+2\operatorname{erf}\{\frac{x}{2\sqrt{\gamma(t-\tau)}}\}\right]Q(\tau)d\tau \qquad [3]$$

RETURN FLOW RATE

The return flow response per unit length of river reach (from one side of the river) due to a uniform unit impulse withdrawal rate per unit area is obtained as usual (e.g., Morel-Seytoux, 1979, p.53) by calculating the flux of water across the saturated thickness at river bank (x=0), namely:

$$- T \frac{\partial k_{s,q}(x,t)}{\partial x}\Big|_{x=0} = k_{q_r,q}(t) = - \sqrt{\frac{\gamma}{\pi}}\,\frac{(1 - e^{-\frac{a^2}{4\gamma t}})}{\sqrt{t}} \qquad [4]$$

Derivations of this result have been provided previously (Morel-Seytoux, 1977).

<u>Return Flow Rate Due to a General Withdrawal Pattern Over the Irrigated Are</u>

Again use of the convolution equation yields for the return flow along the river reach the expression:

$$Q_r(t) = - \frac{1}{a}\int_o^t \sqrt{\frac{\gamma}{\pi}}\left[\frac{1 - e^{-\frac{a^2}{4\gamma(t-\tau)}}}{\sqrt{t-\tau}}\right] Q(\tau)d\tau \qquad [5]$$

CUMULATIVE RETURN FLOW VOLUME

The cumulative return flow volume up to time t is defined as:

$$W_r = \int_o^t Q_r(\tau)d\tau \qquad [6]$$

The unit impulse response of cumulative return flow is obtained as usual (Morel-Seytoux, 1979, p.58) by integrating Eq. [4] with respect to time, namely:

$$k_{W_r,Q}(t) = -\frac{1}{a}\sqrt{\frac{\gamma}{\pi}}\int_o^t \left(\frac{1 - e^{-\frac{a^2}{4\gamma\tau}}}{\sqrt{\tau}}\right) d\tau \qquad [7]$$

or equivalently:

$$k_{W_r,q}(t) = -L_r\sqrt{\frac{\gamma}{\pi}}\int_o^t \left(\frac{1 - e^{-\frac{a^2}{4\gamma\tau}}}{\sqrt{\tau}}\right) d\tau \qquad [8]$$

SIMPLIFICATION IN NOTATIONS

In general drawdown will be evaluated only at a characteristic distance from the river where drawdown is roughly the average drawdown in the aquifer below the irrigated area. The unit impulse given by Eq. [1] when evaluated at that abscissa is denoted simply:

$$k_s(t) = \frac{1}{2\phi}\left\{\mathrm{erf}\left(\frac{a-x}{2\sqrt{\gamma t}}\right) - \mathrm{erf}\left(\frac{a+x}{2\sqrt{\gamma t}}\right) + 2\mathrm{erf}\left(\frac{x}{2\sqrt{\gamma t}}\right)\right\} \qquad [9]$$

where x has a particular value (e.g., $\frac{a}{2}$) and the excitation is that per unit area. Thus the representative (average) drawdown at the selected abscissa is given by the equation:

$$\bar{s}(t) = \int_0^t k_s(t-\tau)q(\tau)d\tau = \frac{1}{A}\int_0^t k_s(t-\tau)Q(\tau)d\tau \qquad [10]$$

Similarly the unit impulse kernel of return flow rate is simply denoted $k_r(t)$ and defined by the equation:

$$k_r(t) = -\frac{1}{a}\sqrt{\frac{\gamma}{\pi}}\,\frac{(1 - e^{-\frac{a^2}{4\gamma t}})}{\sqrt{t}} \qquad [11]$$

and the return flow rate is given in general by the expression:

$$Q_r(t) = \int_0^t k_r(t-\tau)Q(\tau)d\tau \qquad [12]$$

SUMMARY OF FORMULAE

Drawdown Unit Impulse Kernel Due to Uniform Withdrawal Rate per Unit Area

$$k_s(t) = \frac{1}{2\phi}\{\mathrm{erf}(\frac{a-x}{2\sqrt{\gamma t}}) - \mathrm{erf}(\frac{a+x}{2\sqrt{\gamma t}}) + 2\mathrm{erf}(\frac{x}{2\sqrt{\gamma t}})\} \qquad [13]$$

Representative (mean) Drawdown Due to a General Pattern of Withdrawal

$$\bar{s}(t) = \frac{1}{A}\int_0^t k_s(t-\tau)Q(\tau)d\tau \qquad [14]$$

Return Flow Rate Unit Impulse Kernel Due to Uniform Withdrawal Rate Per Unit Area

$$k_r(t) = -\frac{1}{a}\sqrt{\frac{\gamma}{\pi}}\,\frac{(1 - e^{-\frac{a^2}{4\gamma t}})}{\sqrt{t}} \qquad [15]$$

<u>**Return Flow Rate from River Reach Due to a General Pattern of Withdrawal**</u>

$$Q_r(t) = \int_0^t k_r(t-\tau)Q(\tau)d\tau \qquad [16]$$

REFERENCES

Morel-Seytoux, H. J. 1977. "Natural Redistribution of Water Table Levels in a One-Dimensional Semi-Infinite Aquifer," Handout No. 14, Class Notes for Course CE 632 Optimal Ground Water Management, Department of Civil Engineering, Colorado State University, Fall 1977, 6 pages.

Morel-Seytoux, H. J. 1979. "Cost-Effective Methodology for Stream-Aquifer Interaction Modeling and Use in Management of Large Scale Systems," HYDROWAR Program, Colorado State University, Fort Collins, CO 80523, December 1979, 75 pages.

PART 2

FORMULATION OF THE MINIMIZATION PROBLEM OF COST OPERATIONS FROM PUMPING AND SURFACE WATER DIVERSIONS FOR IRRIGATION

INTRODUCTION

For a system already constructed (i.e., with existing reservoirs, canals, wells, etc.) and capable of delivering enough water from combined surface and underground supply to meet crop need for optimal crop yield, the maximization of profits from water use is simply obtained by minimization of the cost of water acquisition. In this case the income from the sale of the crops is fixed. It is the sum of the products of price by optimal yield for the various crops. Generally the cost of groundwater is greater than that of surface water as a result of the energy cost for lifting the water. Thus profit can be maximized by minimizing the cost of water. This is achieved by using the right amount of surface and ground waters at the right time in order not to drawdown the aquifer too much. This is of course accomplished by using the surface supply whenever available and groundwater only as a supplemental source. However surface water availability may be limited by the demand of senior downstream surface water rights. The minimization problem arises as a result of such constraints.

WATER COSTS

Diversion amounts per unit time will be expressed either as discharges or velocities (i.e., depths per unit time, which is volume per unit area per unit time). The function D(t) represents the diversion rate (expressed as depth per unit time) from the stream at a diversion point upstream of the irrigation area. Figure 1 displays the

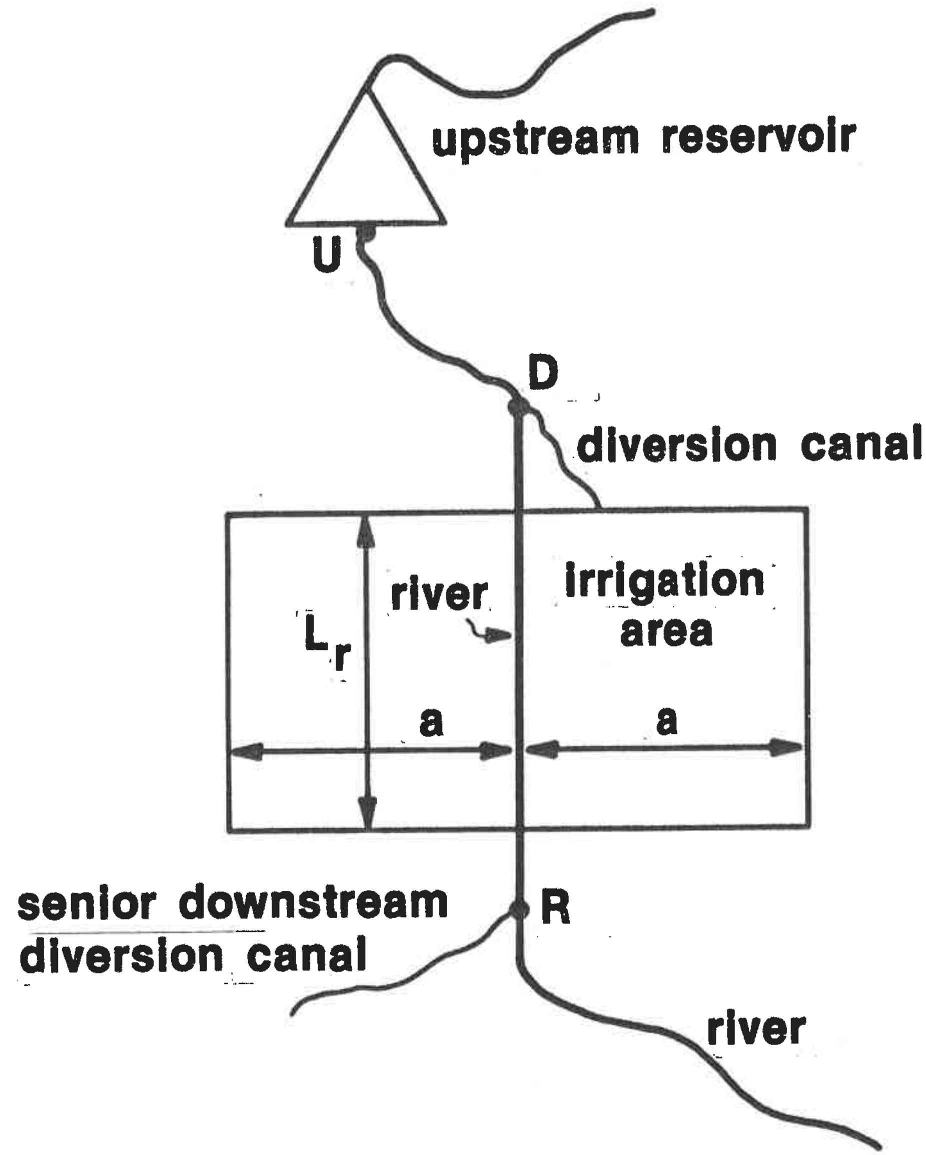

Figure 1. Overall Configuration of System with Upstream Reservoir, Stream, Irrigation Area and Senior Downstream Diversion

overall configuration of the system. Water is released at a rate (velocity) $x(t)$ at point U (upstream point of system of interest). Without any loss this flow reaches point D (diversion point) where a certain amount $D(t)$ (velocity) is diverted. The remaining flow in the river (i.e., $x(t)-D(t)$) will then continue through the irrigation area. Through the reach of length L_r the river is in hydraulic connection with the aquifer. As a result the outgoing flow rate at point R will have increased (algebraically) by the return flow for the reach, $q_r(t)$ expressed as a velocity. Naturally all discharges are converted to velocities by dividing them by the total irrigation area, A.

If c_s denotes the unit cost of surface water diversion then instantaneous cost of diversion is $c_s D(t) A$ and the total cost over the irrigation season is its integral over the irrigation season. The unit cost c_s does not vary within the season (an assumption).

The cost of groundwater is more complex, as it depends on the lift. Drawdown being measured from the initial position of the water table at beginning of irrigation season, used as origin of time, the lifting cost depends on the total lift, which is the initial lift plus the additional lift due to further drawdown during the season. If c_o represents the initial unit cost of pumping and c_m the marginal cost of pumping (i.e., cost per unit pumped volume per additional unit of drawdown) then the total groundwater cost during the irrigation season of duration T plus the total surface diversion cost, is given by the expression:

$$Z = \int_0^T \left[c_o + c_m \bar{s}(t)\right] Q(t)dt + \int_0^T c_s AD(t)dt \qquad [1]$$

where $\bar{s}(t)$ is a representative drawdown for the area and $Q(t)$ is the pumped discharge. The drawdown $\bar{s}(t)$ being a linear function of pumped discharges, it is clear from Eq. [1] that the total water cost will be a quadratic function of pumping rates and a linear one of surface diversion rates.

CONSTRAINTS

There are limitations to the availability of water from the surface reservoir. Denoting by X_T the total available volume of water for the season from the reservoir per unit of irrigation area then clearly the total volume of release cannot be greater than that amount. However, because cost of surface water is relatively cheap, that total volume will indeed be used. Consequently the constraint takes the form:

$$\int_o^T x(t)dt = X_T \tag{2}$$

where T is the duration of the irrigation season. There is a downstream water right which is a total volumetric right for the season, denoted W_T when expressed per unit area of irrigation. Consequently since river outflow from the irrigation area is instantaneously $x(t)-D(t) + q_r(t)$, the mathematical expression of the required total satisfaction of water right is:

$$\int_o^T \left[x(t) - D(t) + q_r(t)\right]dt = W_T \tag{3}$$

The equality is justified by the fact that the upstream users have no interest in losing cheap water to downstream users.

To produce a crop abundantly and of good quality, a proper amount of water has to be delivered to the crop. This amount varies and is denoted $e(t)$ (for evapotranspiration need). This function is a known function of time. Not all the water diverted will reach the plant (in its specific location in a furrow, etc.). Some of it is lost by seepage before it gets to the field. The fraction of diverted water that will actually reach the fields is denoted E_f. Of that amount which reaches the fields only a fraction denoted E_p will actually reach the plant and be transpired. In other words to meet the plant need $e(t)$ an amount $D(t)$ is to be diverted which is $e(t)/E_fE_p$, an amount which can be substantially greater than the plant need. Pumped water can also be used to meet that need. Being withdrawn right on the field, pumped water suffers only one inefficiency. The constraint (requirement) that plant need be met takes the mathematical form:

$$E_fE_pD(t) + E_pq(t) = e(t) \qquad [4]$$

OPTIMIZATION FORMULATION

The optimization problem is one of minimization of the objective function defined by Eq. [1]. This objective function is not fully explicited because $\bar{s}(t)$ is a function of the net withdrawal rate (per unit area). This net withdrawal rate is the difference between pumped rate and aquifer recharge from water application. The net withdrawal rate (velocity) is thus:

$$q_n(t) = q(t) - (1-E_p)q(t) - (1-E_fE_p)D(t) \qquad [5]$$

or defining for simplicity $E_{fp} = E_f E_p$ and $E_r = 1-E_{fp}$, which is the recharge efficiency of the surface diversion:

$$q_n(t) = E_p q(t) - E_r D(t) \tag{6}$$

From the theory of linear systems the drawdown $\bar{s}(t)$ is expressed as a convolution integral:

$$\bar{s}(t) = \int_o^t k_s(t-\tau)\, q_n(\tau)\, d\tau \tag{7}$$

The specific form of the kernel depends upon the representation of the aquifer behavior and its characteristics. For a very simple situation the kernel $k_s(\cdot)$ was derived earlier (see Eq. [13] in Part I). Substitution of Eq. [7] into the objective function transforms the optimization problem in the explicit form:

$$\text{Minimize } \{\int_o^T \{c_o + c_m \int_o^t k_s(t-\tau) \left[E_p q(\tau) - E_r D(\tau)\right]\} q(t)\,dt + c_s \int_o^T D(t)\,dt\} \tag{8}$$

subject to the various constraints defined by Eqs. [2], [3] and [4]. There are three decision functions: $x(t)$, $D(t)$, and $q(t)$. Two of them are not really independent due to constraint Eq. [4]. That equation can be used to express $D(t)$ in terms of $q(t)$ and $e(t)$, namely:

$$D(t) = \frac{e(t)}{E_{fp}} - \frac{q(t)}{E_f} \tag{9}$$

and in turn Eq. [9] can be used to eliminate $D(t)$ from the objective function in Eq. [8]. After substitution the objective function takes the form:

$$z = c_o \int_o^T q(t)dt + c_s \int_o^T D(t)dt + c_m \int_o^T \{\int_o^t k_s(t-\tau) \left[E_p q(\tau) - \frac{E_r e(\tau)}{1-E_r} + \frac{E_r}{E_f} q(\tau)\right] d\tau\} q(t)dt$$

or:

$$z = c_o \int_o^T q(t)dt + c_s \int_o^T D(t)dt + c_m \int_o^T \{\int_o^t k_s(t-\tau) \left[\frac{q(\tau)}{E_f} - f(\tau)\right] d\tau\} q(t)\, dt \qquad [10]$$

where for simplicity the known function $\frac{E_r}{1-E_r} e(\cdot)$ has been redefined, temporarily, as $f(\cdot)$. Thus the optimization of the objective function depends now explicitly on two arbitrary functions: $x(t)$ and $q(t)$. To complete the elimination of $D(t)$ from the objective function its integral has to be rewritten in the form:

$$\int_o^T D(t)dt = \int_o^T \frac{e(t)dt}{E_{fp}} - \int_o^T \frac{q(t)dt}{E_f}$$

Substitution of this expression into the objective function yields:

$$z = (c_o - \frac{c_s}{E_f}) \int_o^T q(t)dt + \frac{c_s}{E_{fp}} \int_o^T e(t)dt +$$

$$c_m \int_o^T \{\int_o^t k_s(t-\tau) \left[\frac{q(\tau)}{E_f} - f(\tau)\right] d\tau\}\; q(t)dt \qquad [11]$$

The problem, once more, is to minimize this objective function with respect to the unknown functions $q(t)$ and $x(t)$ subject to the constraints defined by Eqs. [2] and [3]. After elimination of $D(t)$ from Eq. [3] that constraint takes the form:

$$\int_0^T \{x(t) - \left[\frac{e(t)}{1-E_f} - \frac{q(t)}{E_f}\right] + q_r(t)\}\, dt = W_T$$

or, defining the total depths of evapotranspiration crop need, of water right and of pumpage as:

$$E_T = \int_0^T e(t)dt$$

$$Q_T = \int_0^T q(t)dt$$

and
$$W_T = \int_0^T w(t)dt$$

where w(t) is the downstream surface water right rate (velocity), finally:

$$- E_f \int_0^T q(t)dt - \frac{1}{E_f} \int_0^T \{k_r(t-\tau)q(\tau)d\tau\}dt = X_T - W_T$$

$$- \frac{1}{1-E_r} E_T - \int_0^T \{\int_0^t k_r(t-\tau)f(\tau)d\tau\}dt \qquad [12]$$

Note that the kernel of return flow due to withdrawal is a negative function so that the second term on the left hand side is actually positive. The same comment applies for the last term on the right hand side.

FORMULATION SUMMARY

The minimization problem involves the objective function:

$$z = (c_o - \frac{c_s}{E_f}) \int_0^T q(t)dt + \frac{c_s}{E_f E_p} \int_0^T e(t)dt + c_m \int_0^T \{\int_0^t k_s(t-\tau)\left[\frac{q(\tau)}{E_f} - \frac{E_r e(\tau)}{1-E_r}\right]d\tau\} q(t)dt \qquad [13]$$

and the equality constraint:

$$X_T - W_T - \frac{E_T}{1-E_r} + E_f \int_0^T q(t)dt + \int_0^T \{\int_0^t k_r(t-\tau)\left[\frac{q(\tau)}{E_f} - \frac{E_r e(\tau)}{1-E_r}\right]d\tau\}dt = 0 \quad [14]$$

There are two unknown functions, $q(t)$ and $x(t)$, but only one appears in Eqs. [13] and [14]. The problem appears to be one in classical Calculus of Variations. In order to discover (hopefully) a general method of solution a simple limiting case will be investigated first.

SIMPLE SPECIAL CASE:

Let us assume that the cost of surface water is very cheap (i.e., $c_s = 0$ for practical purposes), that initial cost of pumping is very small (i.e., $c_o = 0$), and that efficiencies E_f and E_p are both one. Then in this case the objective function reduces to:

$$z_s = c_m \int_0^T \{\int_0^t k_s(t-\tau)q(\tau)d\tau\}q(t)dt \quad [15]$$

and the equality constraint reduces to:

$$X_T - W_T - E_T + \int_0^T q(t)dt + \int_0^T \{\int_0^t k_r(t-\tau)q(\tau)d\tau\}dt = 0 \quad [16]$$

One possible strategy of operation is not to pump at all. In that case the cost is minimal (zero). However such strategy is feasible only if $X_T - W_T - E_T \geq 0$ that is if the irrigation requirement E_T and W_T can both be met by seasonal surface storage availability X_T. If such volume is not sufficient then the need will have to be supplied by depleting the aquifer somewhat. Thus in the situation of a deficit in surface water availability (i.e., $X_T < E_T + W_T$) it will be necessary to draw water from the aquifer. In order to get a feeling about the problem and its solution, let us consider the effect of different strategies: for example, withdrawal of full pumping need at beginning of season, at the end of the season or continuously throughout the season.

Full Pumping Need Taken at End of Season

In this case the withdrawal is a unit impulse of magnitude Q_T (expressed as a depth). The objective function of Eq. [15] becomes:

$$z_{se} = c_m \int_o^T \{\int_o^t k_s(t-\tau) Q_T D_\delta(T-\tau) d\tau\} Q_T D_\delta(T-t) dt \qquad [17]$$

(The subscript e refers to the strategy of pumping at end of season) where $D_\delta(\cdot)$ is the Dirac delta function singular at time T. The inner integral is zero except at $\tau=t=T$, where it takes the value $\frac{1}{2}k_s(o)Q_T$, and the total pumping cost is:

$$z_{se} = \frac{1}{2}\, c_m k_s(o) Q_T^2 = \frac{c_m}{2\phi}\, Q_T^2 \qquad [18]$$

In this case $Q_T = E_T + W_T - X_T$ so that the total cost is explicitly:

$$z_{se} = \frac{c_m}{2\phi}\, (E_T + W_T - X_T)^2 \qquad [19]$$

Full Pumping Need Taken at Beginning of Season

The withdrawal is a unit impulse of magnitude Q_T but occurring at time 0. The objective function of Eq. [15] takes the form:

$$z_{sb} = c_m \int_o^T \{\int_o^t k_s(t-\tau)Q_T D_\delta(\tau)d\tau\} Q_T D_\delta(t)dt$$

or

$$z_{sb} = c \int_o^T \frac{1}{2} Q_T k_s(t) Q_T D_\delta(t)dt \qquad [20]$$

or

$$z_{sb} = \frac{c_m}{2} k_s(o) Q_T^2 = \frac{c_m Q_T^2}{2\phi} \qquad [21]$$

(The subscript b refers to the strategy of pumping at beginning of season.) Q_T is now given by the constraint equation [16], namely:

$$X_T - W_T - E_T + Q_T + \int_o^T Q_T k_r(t)dt = 0$$

or

$$Q_T = \frac{E_T + E_T - X_T}{1 + \int_o^T k_r(t)dt} \qquad [22]$$

Since $k_r(\cdot)$ is a negative function Q_T in Eq. [22] exceeds the strict need $E_T + W_T - X_T$. The cost in this strategy of early pumping is larger than for the case of pumping at the last minute. Naturally pumping at the last minute is not a feasible strategy because the crop need e(t) must be satisfied at all times. Similarly the early pumping strategy is not feasible unless the water is stored and delivered as needed during the season.

Use of Groundwater After Exhaustion of Surface Water Supply (last resort)

Until the time t_o such that:

$$\int_o^{t_o} e(t)dt = X_T - W_T \qquad [24]$$

then clearly (?) the optimal policy is (might be) to meet the crop need from the reservoir release and by diversion of surface water. Past time t_o one must pump from the aquifer but then again just to meet the need or in this case:

$$q(t) = e(t) \qquad \text{for} \qquad t_o \leq t \leq T \qquad [25]$$

The minimum cost will be attained for a value:

$$z_{s1} = c_m \int_{t_o}^{T} \{\int_{t_o}^{t} k_s(t-\tau)\ e(\tau)d\tau\}\ e(t)dt \qquad [26]$$

(The subscript 1 refers to fact that groundwater is used as last resort.) However, one constraint, Eq. [16] is not satisfied, because the last term introduces a lack of balance, namely the integral:

$$\int_{t_o}^{T} \{\int_{t_o}^{t} k_r(t-\tau)\ e(\tau)d\tau\}\ dt$$

Enough surface water is available to meet the downstream water right in the interval t_o to T but not to compensate for the seepage induced by pumping.

Pumping as Last Resort but with Continuous Satisfaction of Water Right

The water right function is actually usually defined as a rate $w(t)$. The continuous (permanent) satisfaction of the water right requires that:

$$x(t) - D(t) + q_r(t) = w(t)$$

A strategy that would meet irrigation need and water right without pumping until surface storage is exhausted will dry the river beyond that point. If drying of the river is not acceptable, which will now be assumed, pumping will have to occur before the seasonal surface storage availability is depleted. The time of initiation of pumping t_p is now an unknown. Until the time t_p the strategy is to release water to meet consumptive use and water right, that is:

$$x(t) = e(t) + w(t) \qquad o < t \leq t_p \qquad [28]$$

Let the integrals of $x(\cdot)$, $e(\cdot)$ and $w(\cdot)$ up to that time t_p be denoted X_p, E_p and W_p. Beyond that time the reservoir release is used solely to meet the water right and to compensate for the seepage rate induced by pumping, that is:

$$x(t) = w(t) - q_r(t) \qquad t_p \leq t \leq T \qquad [29]$$

whereas the pumping rate is determined by the consumptive use requirement, namely:

$$q(t) = e(t) \qquad t_p \leq t \leq T \qquad [30]$$

The return flow $q_r(t)$ is related to the pumping rate, in this case e(t), by the relation:

$$q_r(t) = \int_{t_p}^{t} k_r(t-\tau)e(\tau)d\tau \qquad [31]$$

Substitution in Eq. [29] yields the explicit constraint:

$$x(t) = w(t) - \int_{t_p}^{t} k_r(t-\tau)e(\tau)d\tau \qquad [32]$$

The objective function to be minimized is:

$$z_{sc} = c_m \int_{t_p}^{T} \{\int_{t_p}^{t} k_s(t-\tau)e(\tau)d\tau\}\ e(t)dt \qquad [33]$$

(The subscript c refers to the fact that water right is satisfied <u>c</u>ontinuously.) The problem is reduced to one of minimization with respect to one unknown parameter t_p. Redefining the origin of time at the beginning of pumping and the pumping duration time $T-t_p$ as T_p, then Eq. [33] takes the slightly simpler form:

$$z_{sc} = c_m \int_{o}^{T_p} \{\int_{o}^{t} k_s(t-\tau)e(\tau)d\tau\}\ e(t)dt \qquad [34]$$

The minimization of Eq [34] for T_p is subject to the constraint over the irrigation season that:

$$X_T - W_T - E_T\, Q_T + \int_{o}^{T_p} \{\int_{o}^{t} k_r(t-\tau)e(\tau)\}dt = 0 \qquad [35]$$

Defining for convenience the excess need over seasonal storage water availability, namely $E_T+W_T-X_T$ as N_T, Eq. [35] takes the form:

$$\int_0^{T_p}\left[e(t) + \int_0^t k_r(t-\tau)e(\tau)d\tau\right]dt = N_T \qquad [36]$$

Actually Eq. [36] determines T_p since $k_r(\cdot)$, $e(\cdot)$ and N_T are given. Then once T_p is calculated from Eq. [36], substitution of the numerical value of T_p in Eq. [34] yields the value of pumping cost for the season. Prior to time $t_p = T-T_p$ the release rate is given by Eq. [28] and after t_p it is given by Eq. [32]. The diversion rate is $e(t)$ before t_p and zero afterward.

<u>Pumping as Supplement to Surface Diversion with Continuous Satisfaction of Water Right</u>

In the previous strategy need was met solely by surface water up to initiation of pumping and thereafter solely by pumping. An alternative (more general) would initiate pumping while surface diversion continues. It is rather intuitively clear that such a strategy would induce seepage from river earlier and consequently require a larger fraction of X_T to meet downstream water rights. A smaller fraction of X_T would be used for irrigation and as a result a greater pumped volume would be required to meet the consumptive use. Altogether the strategy would cost more. Nevertheless it is instructive to consider this strategy. In this case the release rate is related to need, water right and pumping by the relation:

$$x(t) = e(t) + w(t) - q(t) - \int_0^t k_r(t-\tau)q(\tau)d\tau \qquad [37]$$

whereas the objective still is:

$$z_{ss} = c_m \int_o^T \{\int_o^t k_s(t-\tau)q(\tau)d\tau\}\ q(t)dt \qquad [38]$$

(The second subscript s refers to the fact that in this strategy groundwater is used as a supplement not entire replacement for surface water.) The global form of Eq. [37] for the irrigation season is written more generally:

$$\int_o^T q(t)dt + \int_o^T (\int_o^t k_r(t-\tau)q(\tau)d\tau)dt \geq N_T$$

This inequality expresses the fact that the downstream flow must meet or exceed the water right. The inequality may be rewritten in the form:

$$-\int_o^T q(t)dt - \int_o^T (\int_o^t k_r(t-\tau)q(\tau)d\tau)dt + N_T \leq 0 \qquad [39]$$

which is the standard form to express the constraint to write the Lagrangian function (for example to derive the Kuhn-Tucker theorem). In particular it is known that at the minimum the Lagrange multiplier λ is positive or zero.

Fundamentally the problem is to minimize the objective given by Eq. [38] for the unknown function q(t) subject to Eq. [39]. The Lagrangian function associated with the objective function in Eq. [38] is:

$$L = c_m \int_o^T \{\int_o^t k_s(t-\tau)q(\tau)d\tau\}\ q(t)dt$$

$$+ \lambda \left[- \int_o^T q(t)dt - \int_o^T \{\int_o^t k_r(t-\tau)q(\tau)d\tau\}\ dt + N_T\right] \qquad [40]$$

It remains to derive the Euler-Lagrange equation for this functional problem. The change in Lagrangian when $q(t)$ changes to $q(t)+\varepsilon\eta(t)$, where $\varepsilon\eta(t)$ represents a variation in $q(t)$, is:

$$\Delta L = c_m \varepsilon \int_o^T \left(\int_o^t k_s(t-\tau) q(\tau) d\tau\right) \eta(t) dt + c_m \varepsilon \int_o^T \left(\int_o^t k_s(t-\tau) \eta(\tau) d\tau\right) q(t) dt$$

$$- \lambda\varepsilon \int_o^T \eta(t) dt - \lambda\varepsilon \int_o^T \left(\int_o^t k_r(t-\tau) \eta(\tau) d\tau\right) dt + O(\varepsilon^2) \qquad [41]$$

If the function $q(t)$ is to minimize L then the coefficient of ε must be zero for all arbitrary $\eta(t)$. After interchange of order of integration in the second and fourth integral in Eq. [41] one obtains.

$$\Delta L = c_m \varepsilon \int_o^T \left(\int_o^t k_s(t-\tau) q(\tau) d\tau\right) \eta(t) dt$$

$$+ c_m \int_o^T \left(\int_\tau^T k_s(t-\tau) q(t) dt\right) \eta(\tau) d\tau$$

$$- \lambda\varepsilon \int_o^T \eta(t) dt - \lambda\varepsilon \int_o^T \left(\int_\tau^T k_r(t-\tau) dt\right) \eta(\tau) d\tau + O(\varepsilon^2) \qquad [42]$$

Changing the name of the time variables in the second and fourth integral yields:

$$\Delta L = \varepsilon \int_o^T \{c_m \int_o^t k_s(t-\tau) q(\tau) d\tau + c_m \int_t^T k_s(\tau-t) q(\tau) d\tau$$

$$- \lambda - \lambda \int_t^T k_r(\tau-t) d\tau\} \; \eta(t) dt \qquad [43]$$

The Euler-Lagrange equation is thus:

$$c_m\left[\int_o^t k_s(t-\tau)q(\tau)d\tau + \int_t^T k_s(\tau-t)q(\tau)d\tau\right] = \lambda(1 + \int_t^T k_r(\tau-t)d\tau) \qquad [44]$$

It is an integral equation for the unknown function $q(\cdot)$. Eq. [44] can be expressed in a more standard form by defining a kernel:

$$k_s^*(u) = k_s(|u|) \qquad [45]$$

and by taking $T = \infty$ in the second irrigation (beyond the real irrigation season then $e(t) = 0$ and $q(t) = 0$). Eq. [44] takes the form:

$$c_m \int_o^\infty k_s^*(t-\tau)q(\tau)d\tau = \lambda \ (1+ \int_t^T k_r(\tau-t)d\tau) \qquad [46]$$

which is a linear integral equation of the first kind. The solution is a function of the (unknown) Lagrange multiplier λ. This multiplier is then found by substitution of the solution $q(t,\lambda)$ into the constraint Eq. [39], taken as an equality, which becomes an algebraic equation to be solved for λ. Once λ is obtained substitution of its value into the solution $q(t,\lambda)$ yields the optimal solution $q^*(t)$ to the problem.

However the solution so obtained is not valid if $q(t)$ is < 0 or $> e(t)$, since clearly 0 and $e(t)$ are bounds for the pumping rate in the supplemental strategy.

Suppose that the optimal solution was on the lower bound constraint at time t (thus $q(t)=0$). The only feasible variation is $\eta(t) > 0$. If indeed the objective is at a minimum then ΔL has to be positive for a positive variation $\eta(t)$. It follows that if $q=0$ is optimal in the interval $(0,t_p)$ the coefficient of $\varepsilon(t)$ in that range in Eq. [43] has to be positive namely:

$$\int_o^t k_s(t-\tau)q(\tau)d\tau + \int_t^T k_s(\tau-t)q(\tau)d\tau \geq \frac{\lambda}{c_m}(1+\int_t^T k_r(\tau-t)d\tau)$$

or more precisely since q is zero in interval $(0,t_p)$

$$\int_{t_p}^T k_s(\tau-t)q(\tau)d\tau \geq \frac{\lambda}{c_m}(1+\int_t^T k_r(\tau-t)d\tau) \quad \text{for } 0 \leq t \leq t_p \qquad [47]$$

Since λ is positive, this equation implies that q=0 up to time t_p can be optimal provided that beyond that time pumping is large enough and/or that t_p is small (i.e., pumping is initiated early) and/or that c_m is large. Similarly one may question whether or not q(t) = e(t) can be an optimal policy. Suppose that the optimal solution was on the upper bound for times $t_e \leq t \leq T$. In that range the only feasible variation is $\eta(t) < 0$. The coefficient of $\eta(t)$ in that range of times has to be negative, thus:

$$\int_o^{t_e} k_s(t-\tau)q(\tau)d\tau + \int_{t_e}^t k_s(t-\tau)e(\tau)d\tau$$
$$+ \int_{t_e}^T k_s(\tau-t)e(\tau)d\tau \leq \frac{\lambda}{c_m}(1+\int_t^T k_r(\tau-t)d\tau) \quad \text{for } t_e \leq t \leq T \qquad [48]$$

Since λ is positive Eq. [48] implies that t_e cannot be too small and/or that prior to t_e q(t) must be small and/or that c_m is small. Eqs. [47] and [48] imply that at early times a solution q=0 is optimal and that at late times q=e is optimal. There remains a question about the possibility of an optimal q in the range (0,e) during the interval (t_p,t_e). In that case Eq. [44] must hold for $t_p \leq t \leq t_e$.

This discussion provides a basis to check whether the "bang-bang" solution that is q=0 up to t_p and q=e thereafter is indeed optimal. The solution for t_p in this strategy is given by Eq. [36] or more precisely by:

$$\int_{t_p}^{T} \left[e(t) + \int_{t_p}^{T} k_r(t-\tau)e(\tau)d\tau \right] dt = N_T \qquad [49]$$

Having determined t_p one would next verify that Eq. [47] holds, in this case:

$$\int_{t_p}^{T} k_s(\tau-t)e(\tau)d\tau \geq \frac{\lambda}{c_m}\left(1+\int_{t}^{T} k_r(\tau-t)d\tau\right)$$

$$\text{for} \quad 0 \leq t \leq t_p \qquad [50]$$

The value of λ is determined from Eq. [46] for q(t) being a step function jumping from zero to $e(t_p)$ at $t=t_p$, namely:

$$\lambda = \frac{c_m \int_{t_p}^{T} k_s(\tau-t_p)e(\tau)d\tau}{1 + \int_{t_p}^{T} k_r(\tau-t_p)d\tau} \qquad [51]$$

One would also need to verify that Eq. [48] holds, in this case:

$$\int_{t_p}^{t} k_s(t-\tau)e(\tau)d\tau + \int_{t}^{T} k_s(\tau-t)e(\tau)d\tau \leq \frac{\lambda}{c_m}\left(1+\int_{t}^{T} k_r(\tau-t)d\tau\right. \qquad [52]$$

From the value of λ in Eq. [51], Eq. [50] takes the more specific form:

$$\frac{\int_{t_p}^{T} k_s(\tau-t)e(\tau)d\tau}{1+\int_{t}^{T} k_r(\tau-t)d\tau} \geq \frac{\int_{t_p}^{T} k_s(\tau-t_p)e(\tau)d\tau}{1+\int_{t_p}^{T} k_r(\tau-t_p)d\tau}$$

$$\text{for} \quad 0 \leq t \leq t_p \tag{53}$$

Similarly Eq. [52] takes the form:

$$\frac{\int_{t_p}^{t} k_s(t-\tau)e(\tau)d\tau + \int_{t}^{T} k_s(\tau-t)e(\tau)d\tau}{1+\int_{t}^{T} k_r(\tau-t)d\tau} \leq \frac{\int_{t_p}^{T} k_s(\tau-t_p)e(\tau)d\tau}{1+\int_{t_p}^{T} k_r(\tau-t_p)d\tau}$$

$$\text{for} \quad t_p \leq t \leq T \tag{54}$$

It is not possible to state whether the "bang-bang" solution is the optimal one in all situations. The satisfaction of Eqs. [53] and [54] depends upon the shape of the kernels $k_s(\cdot)$ and $k_r(\cdot)$ and of the crop need $e(\cdot)$.

Consider the simpler case when the irrigation area extends far from the river. In that case the return flow kernel has the form:

$$k_r(t) = -\frac{1}{a}\sqrt{\frac{\gamma}{\pi}}\,\frac{1}{\sqrt{t}} \tag{55}$$

In particular its integral with respect to time (the unit step kernel) is:

$$K_r(t) = -\frac{1}{a}\sqrt{\frac{\gamma}{\pi}}\int_{o}^{t}\frac{1}{\sqrt{\tau}}\,d\tau = -\frac{2}{a}\sqrt{\frac{\gamma}{\pi}}\,\sqrt{t}$$

Consequently:

$$\int_{t}^{T} k_r(\tau-t)d\tau = K_r\ (T-t) = -\frac{2}{a}\sqrt{\frac{\gamma}{\pi}}\ \sqrt{T-t}$$

whereas:

$$\int_{t_p}^{T} k_r(\tau-t_p)d\tau = K_r(T-t_p) = -\frac{2}{a}\sqrt{\frac{\gamma}{\pi}}\ \sqrt{T-t_p}$$

For $t \leq t_p$ then it follows that the denominator of the left-hand side of Eq. [53] is less than that on the right-hand side. Everything else the same, the larger the seepage flow (that is the better the hydraulic connection between stream and aquifer) the longer one waits to pump to operate optimally.

Similarly again for the case of an area extending far from the river the drawdown kernel has the form:

$$k_s(t) = \frac{1}{\phi}\ \mathrm{erf}\ \left(\frac{x}{2\sqrt{\gamma t}}\right) \qquad [56]$$

Supposing a constant consumptive use e(t) then the numerator on the left-hand side of Eq. [53] is proportional to:

$$\int_{t_p}^{T} \mathrm{erf}\left[\frac{x}{2\sqrt{\gamma(\tau-t)}}\right]d\tau = \frac{x^2}{2\gamma}\int_{\frac{x^2}{2\gamma(T-t)}}^{\frac{x^2}{2\gamma(t_p-t)}} \mathrm{erf}\ (u)\ \frac{du}{u^2}$$

whereas on the right-hand side the numerator is proportional to:

$$\int_{\frac{x^2}{2\gamma(T-t_p)}}^{\infty} \mathrm{erf}\ (u)\ \frac{du}{u^2}$$

The main contribution to these integrals comes from the lower limit. Thus for constant e the numerator on the left-hand side tends to be greater than the numerator on the right-hand side. The discussion tends to indicate that in many situations the bang-bang policy will be optimal but it is not sure.

General Procedure (for still the simple case)

One presumes that the "bang-bang" policy is optimal. The value of t_p is determined from Eq. [49]. One then checks that Eq. [53] is satisfied for $t \leq t_p$. One checks that Eq. [54] is satisfied for all $t > t_p$. If the checks are positive then the optimal solution was obtained. In the negative one must relax the assumption that at initiation of pumping pumping rate takes immediately the value of irrigation need. At this stage an iterative procedure becomes necessary. Selecting values of t_p and t_e a priori one solves Eq. [44] for values of t in the interval (t_p, t_e), more specifically:

$$c_m \left[\int_{t_p}^{t} k_s(t-\tau) q(\tau) d\tau + \int_{t}^{t_e} k_s(\tau-t) q(\tau) d\tau \right]$$
$$= - c_m \int_{t_e}^{T} k_s(\tau-t) e(\tau) d\tau + \lambda(1 - \int_{t}^{T} k_r(\tau-t) d\tau) \qquad [57]$$

The solution of this integral equation depends upon λ. Substitution of this solution for $q(t,\lambda)$ in Eq. [39] leads more specifically to the expression:

$$N_T - \int_{t_p}^{t_e} q(t,\lambda) dt - \int_{t_e}^{T} e(t) dt - \int_{t_p}^{T} (\int_{t_p}^{t} k_r(t-\tau) q(\tau,\lambda) d\tau) dt = 0 \qquad [58]$$

Once λ obtained one proceeds to Eqs. [47] and [48] for checks on optimality. If the tests are positive the solution has been found. In the negative one must reestimate t_p and t_e, etc.

CONCLUSIONS

The classical techniques for optimization of decision functions such as pumping rates, release rates, etc., are not powerful enough to find the optimal patterns as continuous functions of time as they really are. Instead the unknown functions are discretized over the time horizon. Often in addition to discretization, simplifications are made in the dependence of the objective function on the decision functions. In particular, as in Dynamic Programming, the instantaneous objective function cannot have a memory dependence on previous decisions. Yet this is precisely the case when there is interaction between stream and aquifer.

In this study it was decided to take a crack at the problem from a Functional Optimization point of view. Because of the intrinsic memory of the cost function on past decisions, not surprisingly the Euler-Lagrange equation turns out to be an integral equation rather than a differential equation (the classical case and only one discussed in the mathematical literature). In the simple case considered for which the Euler Lagrange equation was derived, the optimality condition has a clear economic meaning. Under optimal operations at any given time the marginal capitalized cost of future extra lifts due to additional unit of pumped water at that time equals the immediate marginal penalty cost for failing to meet the downstream legal right by one unit at the same time. Based on this optimality criterion optimal release and pumping decisions can be taken continuously throughout the season of operations. Unfortunately analytical solution of an integral equation, even a linear one, is not an easy task. In fact exact solutions are rare. However there are efficient numerical techniques of solution. Lack of time and other commitments did not permit to explore this new procedure in a quantitative manner for specific values of parameters for a reach of a river in hydraulic connection with an aquifer, at the present time. This will be done in the future. One must capitalize on a good idea when one encounters one!

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